MTH642

Fluid Mechanics

Handouts Written by Mahar Afaq Safdar Muhammadi
maharafaq789@gmail.com
03494873115

Topic No. 01

What is Fluid?

- Recall from physics that a substance exists in three primary phases.
- Solid, liquid and gas.
- A substance in the liquid or gas phase is referred to as a **fluid**.
- Distinction between a solid and a fluid is made on the basis of the substances ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of the shear stress no matter how small. In solids stress is proportional to strain, but in fluids stress is proportional to strain rate.
- **Stress** is defined as force per unit area and is determined by dividing the force by the area upon which it acts.
- The normal component of the acting on a surface per unit area is called the **normal stress**.
- The tangential component of force acting on a surface per unit area is called the **shear stress**.
- In a fluid at rest, the normal stress is called **Pressure**.
- A fluid at rest is at a state of zero shear stress.

Topic No. 02

Application Areas of Fluid Mechanics

- Fluid mechanics is widely used both in everyday activities and in the design of modern engineering system from vacuum cleaners to supersonic aircraft.
- Fluid mechanics plays a vital role in the human body.
- The heart is constantly pumping blood to all parts of the human body through the arteries and veins, and the lungs are the sites of airflow in alternating directions.
- All artificial hearts, breathing machines and dialysis systems are designed using fluid dynamics
- An ordinary house is in some respects, an exhibition hall filled with applications of fluid mechanics.
- The piping systems of cold water, natural gas and sewage for an individual house and the entire city are designed primarily on the basis of fluid mechanics.
- The same is also true for the piping and ducting network of heating and air-conditioning systems.
- A refrigerator involves tubes through which the refrigerant flows a compressor that pressurizes the refrigerant and two heat exchangers where the refrigerant absorbs and rejects heat.
- Fluid mechanics plays a major role in the design of all these components.
- Even the operation of ordinary faucets is based on fluid mechanics.
- We can also see numerous applications in an automobile.
- All components associated with the transportation of the fuel from the fuel tank to the cylinders-the fuel line fuel pump, fuel injectors or carburetors-as well as the mixing of the fuel and the air in the cylinders. The purging of combustion gases in exhaust pipes are analyzed using fluid mechanics.
- Fluid mechanics is also used in the design of the heating and air-conditioning system, the hydraulic brakes, the power steering, automatic transmission lubrication systems.
- The cooling system of the engine block including the radiator and the water pump, and even the tires.
- The sleek streamlined shape of the recent model cars is the result of efforts to minimize drag by using extensive analysis of flow over surfaces, fluid mechanics plays a major part in the design and analysis of aircraft, boats, submarines, rockets, jet engines, wind turbines, biomedical devices, the cooling of electronic components and the transportation of water, crude oil and natural gas.
- It is also considered in the design of buildings, bridges and even billboards to make sure that the structures can withstand wind loading.

- Numerous natural phenomena such as the rain cycle, weather patterns, the rise of ground water to the top of trees, winds, ocean waves and currents in large water bodies are also governed by the principles of fluid mechanics.

Topic No. 03

The No-slip Condition

- Fluid flow is often confined by solid surfaces and it is important to understand how the presence of solid surfaces affects fluid flow.
- We know that water in a river cannot flow through large rocks and goes around them.
- That is, the water velocity normal to the rock surfaces must be zero and water approaching the surface normally comes to a complete stop at the surface.
- What is not so obvious is that water approaching the rock at any angle also comes to a complete stop at the rock surface and thus the tangential velocity of water at the surface is also zero.
- Consider the flow of a fluid in a stationary pipe or over a solid surface that is nonporous (i.e. impermeable to the fluid) solid "sticks" to the surface due to viscous effects and there is no slip. This is known as the **no slip condition**.
- The layer that sticks to the surface slows the adjacent fluid layer because of viscous forces between the fluid layers, which slows the next layer, and so on.
- Therefore, the no-slip condition is responsible for the development of the velocity profile.
- The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer**.
- The fluid property responsible for the no-slip condition and the development of the boundary layer is viscosity and is discussed later.
- A fluid layer adjacent to a moving surface has the same velocity as the surface.
- A consequence of the non-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface.
- Another consequence of the no-slip condition is the surface drag, which is the force a fluid exerts on a surface in flow direction.
- When a fluid is forced to flow over a curved surface, such as the back side of the cylinder at sufficiently high velocity the boundary layer can no longer remain attached to the surface and at same point it separates from the surface- a process called **flow separation**.
- We emphasize that the no-slip condition applies everywhere along the surface even down-stream of the separation point.
- A similar phenomenon occurs for temperature.

- When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the points of contact. Therefore, a fluid and a solid surface have the same temperature at the points of contact. This is known as no temperature jump condition.

Topic No. 04

A Brief History of Fluid Mechanics-1

- One of the first engineering problems humankind faced as cities were developed was the supply of water for domestic use and irrigation of crops.
- Our urban lifestyles can be retained only with abundant water and it is clear from archeology that every successful civilization of prehistory invested in the construction and maintenance of water systems.
- The roman aqueducts, some of which are still in use, are the best known examples.
- Greek mathematician Archimedes (285–212 BC). He formulated and applied the buoyancy principle in history's first nondestructive test to determine the gold content of the crown of King Hiero II.
- During the Middle Ages, the application of fluid machinery slowly but steadily expanded.
- Elegant piston pumps were developed for dewatering mines, and the watermill and windmill were perfected to grind grain, forge metal, and for other tasks. For the first time in recorded human history, significant work was being done without the power of a muscle supplied by a person or animal, and these inventions are generally credited with enabling the later industrial revolution.
- Again the creators of most of the progress are unknown, but the devices themselves were well documented by several technical writers such as Georgius Agricola.
- Simon Stevin (1548–1617), Galileo Galilei (1564–1642), Edme Mariotte (1620–1684), and Evangelista Torricelli (1608–1647) were among the first to apply the method to fluids as they investigated hydrostatic pressure distributions and vacuums.
- That work was integrated and refined by the brilliant mathematician and philosopher, Blaise Pascal (1623–1662).
- The Italian monk, Benedetto Castelli (1577–1644) was the first person to publish a statement of the continuity principle for fluids.
- Besides formulating his equations of motion for solids, Sir Isaac Newton (1643–1727) applied his laws to fluids and explored fluid inertia and resistance, free jets, and viscosity.

- That effort was built upon by Daniel Bernoulli (1700–1782), a Swiss, and his associate Leonard Euler (1707–1783). Together, their work defined the energy and momentum equations.
- Bernoulli's 1738 classic treatise *Hydrodynamica* may be considered the first fluid mechanics text.
- Finally, Jean d'Alembert (1717–1789) developed the idea of velocity and acceleration components, a differential expression of continuity, and his "paradox" of zero resistance to steady uniform motion over a body.

Topic No. 05

A Brief History of Fluid Mechanics-2

- Antonie Chezy (1718–1798), Louis Navier (1785–1836), Gaspard Coriolis (1792–1843), Henry Darcy (1803–1858), and many other contributors to fluid engineering and theory were students and/or instructors at the schools.
- The physician Jean Poiseuille (1799–1869) had accurately measured flow in capillary tubes for multiple fluids, while in Germany Gotthilf Hagen (1797–1884) had differentiated between laminar and turbulent flow in pipes. In England, Lord Osborne Reynolds (1842–1912) continued that work and developed the dimensionless number that bears his name.
- Similarly, in parallel to the early work of Navier, George Stokes (1819–1903) completed the general equation of fluid motion (with friction) that takes their names.
- Fluid theory was further developed by Irish and English scientists and engineers including in addition to Reynolds and Stokes, William Thomson, Lord Kelvin (1824–1907), William Strutt, Lord Rayleigh (1842–1919), and Sir Horace Lamb (1849–1934).
- These individuals investigated a large number of problems, including dimensional analysis, Irrotational flow, vortex motion, cavitation, and waves.
- Twentieth century brought two monumental developments.
  First, in 1903, the self-taught Wright brothers (Wilbur, 1867–1912; Orville, 1871–1948) invented the airplane.
- Their primitive invention was complete and contained all the major aspects of modern aircraft.
- The Navier–Stokes equations were of little use up to this time because they were too difficult to solve.
- In a pioneering paper in 1904, the German Ludwig Prandtl (1875–1953) showed that fluid flows can be divided into a layer near the walls, the *boundary layer,* where the friction effects are significant, and an outer layer where such effects are negligible and the simplified Euler and Bernoulli equations are applicable. His students, Theodor von Kármán (1881–1963),

Paul Blasius (1883–1970), Johann Nikuradse (1894–1979), and others, built on that theory in both hydraulic and aerodynamic applications.

Topic No. 06

A Brief History of Fluid Mechanics-3

- The existing theories supported a huge expansion of the aeronautical, chemical, industrial, and water resources sectors; each of which pushed fluid mechanics in new directions.
- Fluid mechanics research and work in the late twentieth century were dominated by the development of the digital computer in America.
- The ability to solve large complex problems, such as global climate modeling or the optimization of a turbine blade, has provided a benefit to our society that the eighteenth-century developers of fluid mechanics could never have imagined.
- Where will fluid mechanics go in the twenty-first century and beyond? Frankly, even a limited extrapolation beyond the present would be sheer folly.

Topic No. 07

Classification of Fluid Flows

- There is a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups.
- There are many ways to classify fluid flow problems, and here we present some general categories.

Viscous versus Inviscid Regions of Flow

- When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is quantified by the fluid property viscosity, which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases.
- There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree.
- Flows in which the frictional effects are significant are called **viscous flows**.
- However, in many flows of practical interest, there are regions (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure

forces. Neglecting the viscous terms in such **inviscid flow regions** greatly simplifies the analysis without much loss in accuracy.

Topic No. 08

Internal versus External Regions of Flow

- A fluid flow is classified as being internal or external, depending on whether the fluid flows in a confined space or over a surface.
- The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow**.
- The flow in a pipe or duct is **internal flow** if the fluid is bounded by solid surfaces.
- Water flow in a pipe, for example, is internal flow, and airflow over a ball or over an exposed pipe during a windy day is external flow
- The flow of liquids in a duct is called open-channel flow if the duct is only partially filled with the liquid and there is a free surface.
- The flows of water in rivers and irrigation ditches are examples of such flows.
- Internal flows are dominated by the influence of viscosity throughout the flow field.
- In external flows the viscous effects are limited to boundary layers near solid surfaces and to wake regions downstream of bodies.

Topic No. 09

Compressible versus Incompressible Flow

- A flow is classified as being compressible or incompressible, depending on the level of variation of density during flow.
- Incompressibility is an approximation, in which the flow is said to be **incompressible** if the density remains nearly constant throughout.
- Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow is approximated as incompressible.
- The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible.
- Therefore, liquids are usually referred to as incompressible substances.
- Gases, on the other hand, are highly compressible.
- When analyzing rockets, spacecraft, and other systems that involve high speed gas flows, the flow speed is often expressed in terms of the dimensionless **Mach number** defined as

$$Ma=\frac{v}{c}=\frac{\text{Speed of flow}}{\text{Speed of sound}}$$

where c is the **speed of sound** whose value is 346 m/s in air at room temperature at sea level. A flow is called **sonic** when Ma = 1, **subsonic** when Ma < 1, **supersonic** when Ma > 1, and **hypersonic** when Ma ≫ 1.

- Liquid flows are incompressible to a high level of accuracy, but the level of variation of density in gas flows and the consequent level of approximation made when modeling gas flows as incompressible depends on the Mach number.
- Gas flows can often be approximated as incompressible if the density changes are under about 5 percent, which is usually the case when Ma < 0.3.
- Therefore, the compressibility effects can be neglected at speeds under about 100 m/s.
  Note that the flow of gas is not necessarily a compressible flow.
- Small density changes of liquids corresponding to large pressure changes can still have important consequences.

Topic No. 10

Laminar versus Turbulent Flow

- Some flows are smooth and orderly while others are rather chaotic.
- The highly ordered fluid motion characterized by smooth layers of fluid is called
  laminar.
- The word laminar comes from the movement of adjacent fluid particles together in "laminates."
- The flow of high-viscosity fluids such as oils at low velocities is typically laminar.
- The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called **turbulent**.
- The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the required power for pumping.
- A flow that alternates between being laminar and turbulent is called **transitional**.
- The experiments conducted by Osborne Reynolds in the 1880s resulted in the establishment of the dimensionless **Reynolds number Re** as the key parameter for the determination of the flow regime in pipes.

Natural (or Unforced) versus Forced Flow

- A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated.
- In **forced flow**, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan.
- In **natural flows**, fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of warmer (and thus lighter) fluid and the fall of
cooler (and thus denser) fluid.
- In solar hot-water systems, for example, the thermo siphoning effect is commonly used to replace pumps by placing the water tank sufficiently above the solar collectors.
-

Topic No. 11

Steady versus Unsteady Flow-1

- The terms steady and uniform are used frequently in engineering, and thus it is important to have a clear understanding of their meanings.
- The term **steady** implies no change of properties, velocity, temperature, etc., at a point with time. The opposite of steady is **unsteady**.
- Steady implies no change at a point with time.
Transient terms in N-S equations are zero

$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\partial \rho}{\partial t} = 0$$

  - Unsteady is the opposite of steady.
  - Transient usually describes a starting, or developing flow.
  - Periodic refers to a flow which oscillates about a mean.
  - Unsteady flows may appear steady if "time averaged.

- These meanings are consistent with their everyday use (steady girlfriend, uniform distribution, etc.).
- The terms unsteady and transient are often used interchangeably, but these terms are not synonyms.
- In fluid mechanics, unsteady is the most general term that applies to any flow that is not steady, but **transient** is typically used for developing flows.
- When a rocket engine is fired up, for example, there are transient effects (the pressure builds up inside the rocket engine, the flow accelerates, etc.) until the engine settles down and operates steadily.

Topic No. 12

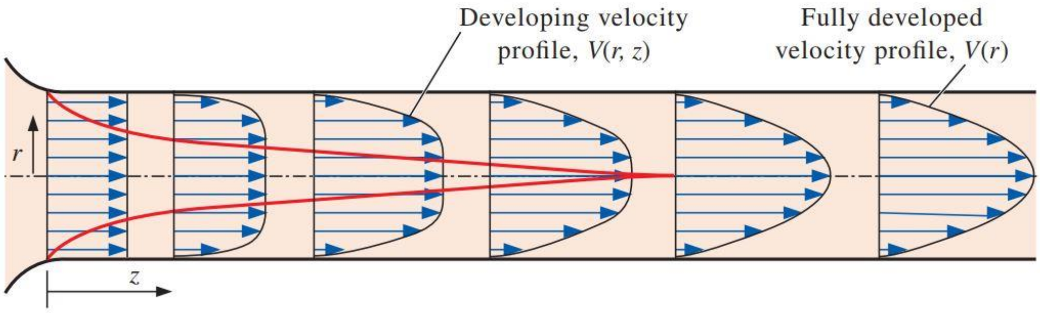
Steady versus Unsteady Flow-2

- The term **periodic** refers to the kind of unsteady flow in which the flow oscillates about a steady mean.
- Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as steady-flow devices.
- (Note that the flow field near the rotating blades of a turbo machine is of course unsteady, but we consider the overall flow field rather than the details at some localities when we classify devices.)
- During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant. Therefore, the volume, the mass, and the total energy content of a steady-flow device or flow section remain constant in steady operation.
- Steady-flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers, and heat exchangers of power plants or refrigeration systems.
- Some cyclic devices, such as reciprocating engines or compressors, do not satisfy the steady-flow conditions since the flow at the inlets and the exits is pulsating and not steady.
- However, the fluid properties vary with time in a periodic manner, and the flow through these devices can still be analyzed as a steady-flow process by using time-averaged values for the properties.
- Most of the analytical and computational examples provided in this textbook deal with steady or time-averaged flows, although we occasionally point out some relevant unsteady-flow features as well when appropriate.

Topic No. 13

One-, Two-, and Three-Dimensional Flows-1

- A flow field is best characterized by its velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three primary dimensions, respectively.
- A typical fluid flow involves a three-dimensional geometry, and the velocity may vary in all three dimensions, rendering the flow three-dimensional [V(x, y, z) in rectangular or V (r, u, z) in cylindrical coordinates].
- However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored with negligible error.



Topic No. 14

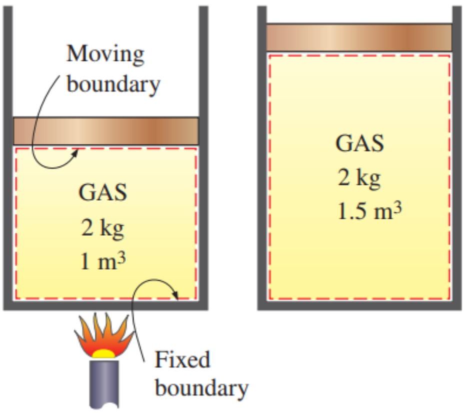
One-, Two-, and Three-Dimensional Flows-2

- In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.
- Consider steady flow of a fluid through a circular pipe attached to a large tank.
- The fluid velocity everywhere on the pipe surface is zero because of the no-slip condition, and the flow is two-dimensional in the entrance region of the pipe since the velocity changes in both the r- and z-directions, but not in the u-direction.
- The velocity profile develops fully and remains unchanged after some distance from the inlet (about 10 pipe diameters in turbulent flow, and typically farther than that in laminar pipe flow), and the flow in this region is said to be fully developed.
- The fully developed flow in a circular pipe is one-dimensional since the velocity varies in the radial r-direction but not in the angular u- or axial z-directions.
- That is, the velocity profile is the same at any axial z-location, and it is symmetric about the axis of the pipe.
- Note that the dimensionality of the flow also depends on the choice of coordinate system and its orientation.
- The pipe flow discussed, for example, is one-dimensional in cylindrical coordinates, but two-dimensional in Cartesian coordinates—illustrating the importance of choosing the most appropriate coordinate system.
- Also note that even in this simple flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition.
- However, at a well-rounded entrance to the pipe, the velocity profile may be approximated as being nearly uniform across the pipe, since the velocity is nearly constant at all radii except very close to the pipe wall.
- A flow may be approximated as two-dimensional when the aspect ratio is large and the flow does not change appreciably along the longer dimension.

Topic No. 15

System and Control Volume-1

- A **system** is defined as a quantity of matter or a region in space chosen for study.
- The mass or region outside the system is called the **surroundings**.
- The real or imaginary surface that separates the system from its surroundings is called the **boundary**.
- The boundary of a system can be fixed or movable. Note that the boundary is the contact surface shared by both the system and the surroundings.
- Mathematically speaking, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.
- Systems may be considered to be closed or open, depending on whether a fixed mass or a volume in space is chosen for study.
- A system is defined as a quantity of matter or a region in space chosen for study.
- A **closed system** consists of a fixed amount of mass.
- An **open system**, or a **control volume**, is a properly selected region in space.

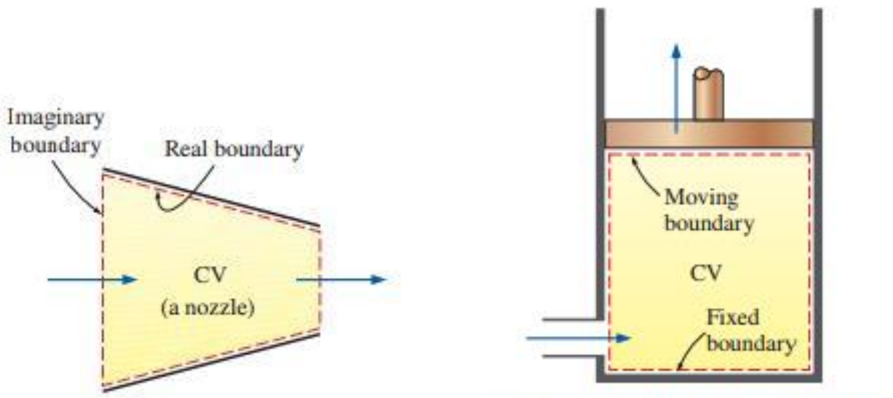


Topic No. 16

System and Control Volume-2

- A **closed system** (also known as a **control mass**) consists of a fixed amount of mass, and no mass can cross its boundary.
- But energy, in the form of heat or work, can cross the boundary, and the volume of a closed system does not have to be fixed.

- If, as a special case, even energy is not allowed to cross the boundary, that system is called an **isolated system**.
- An **open system**, or a **control volume**, as it is often called, is a selected region in space.
- It usually encloses a device that involves mass flow such as a compressor, turbine, or nozzle.
- Flow through these devices is best studied by selecting the region within the device as the control volume.
- Both mass and energy can cross the boundary (the control surface) of a control volume.
- A large number of engineering problems involve mass flow in and out of an open system and, therefore, are modeled as control volumes. A water heater, a car radiator, a turbine, and a compressor all involve mass flow and should be analyzed as control volumes (open systems) instead of as control masses (closed systems).
- In general, any arbitrary region in space can be selected as a control volume.
- There are no concrete rules for the selection of control volumes, but a wise choice certainly makes the analysis much easier.
- If we were to analyze the flow of air through a nozzle, for example, a good choice for the control volume would be the region within the nozzle, or perhaps surrounding the entire nozzle.
- A control volume can be fixed in size and shape, as in the case of a nozzle, or it may involve a moving boundary.
- Most control volumes, however, have fixed boundaries and thus do not involve any moving boundaries.
- A control volume may also involve heat and work interactions just as a closed system, in addition to mass interaction.



(*a*) A control volume (CV) with real and imaginary boundaries

(*b*) A control volume (CV) with fixed and moving boundaries as well as real and imaginary boundaries

Topic No. 17

Importance of Dimensions and Units

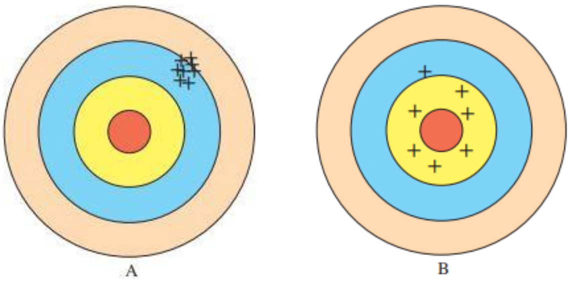
- Any physical quantity can be characterized by **dimensions**. The magnitudes assigned to the dimensions are called **units**.
- Some basic dimensions such as mass m, length L, time t, and temperature T are selected as **primary** or **fundamental dimensions**, while others such as velocity V, energy E, and volume V are expressed in terms of the primary dimensions and are called **secondary dimensions**, or **derived dimensions**.
- A number of unit systems have been developed over the years.
- Despite strong efforts in the scientific and engineering community to unify the world with a single unit system, two sets of units are still in common use today:
- The **English system**, which is also known as the United States Customary System (USCS), and the metric **SI**, which is also known as the International System.
- The SI is a simple and logical system based on a decimal relationship between the various units, and it is being used for scientific and engineering work in most of the industrialized nations, including England.
- The English system, however, has no apparent systematic numerical base, and various units in this system are related to each other rather arbitrarily (12 in = 1 ft, 1 mile = 5280 ft, etc.), which makes it confusing and difficult to learn.
- The United States is the only industrialized country that has not yet fully converted to the metric system.

Topic No. 18

Accuracy, Precision, and Significant Digits

- **Accuracy error** (inaccuracy) is the value of one reading minus the true value. Closeness of the average reading to the true value. Generally associated with repeatable, fixed errors.
- **Precision error** is the value of one reading minus the average of readings. A measure of the fineness of the resolution and the repeatability of them instrument. Generally associated with unrepeatable, random errors.
- **Significant digits** are digits that are relevant and meaningful. When performing calculations or manipulations of several parameters, the final result is generally only as precise as the least precise parameter in the problem.

- When the number of significant digits is unknown, the accepted engineering standard is 3.



- Example of accuracy and precision. Shooter A is more precise, but less accurate, while shooter B is more accurate, but less precise.

Topic No. 19

Fluid Properties

- Any characteristic of a system is called a **property**. Some familiar properties are pressure P, temperature T, volume V, and mass m.
- The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.
- Properties are considered to be either intensive or extensive.
- **Intensive properties** are those that are independent of the mass of the system, such as temperature, pressure, and density.
- **Extensive properties** are those whose values depend on the size—or extent—of the system.
- Extensive properties per unit mass are called **specific properties**. Some examples of specific properties are specific volume ($v = V/m$) and specific total energy ($e = E/m$).

Topic No. 20

Continuum

- Matter is made up of atoms that are widely spaced in the gas phase.
- Yet it is convenient to disregard the atomic nature of the fluid and view it as continuous, homogeneous matter with no holes, that is, a **continuum**.

- The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities.
- This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.
- This is the case in practically all problems, except some specialized ones.
- The continuum idealization is implicit in many statements we make, such as "the density of water in a glass is the same at any point."
- To have a sense of the distances involved at the molecular level, consider a container filled with oxygen at atmospheric conditions.
- The diameter of an oxygen molecule is about 3×10^{-10} m and its mass is 5.3×10^{-26} kg.
- Also, the mean free path λ of oxygen at 1 atm pressure and 20°C is 6.3×10^{-8} m.
- That is, an oxygen molecule travels, on average, a distance of 6.3×10^{-8} m (about 200 times its diameter) before it collides with another molecule.
- At very low pressure, e.g., at very high elevations, the mean free path may become large (for example, it is about 0.1 m for atmospheric air at an elevation of 100 km).
- For such cases the **rarefied gas flow theory** should be used, and the impact of individual molecules should be considered.
- In this course we limit our consideration to substances that can be modeled as a continuum.

Topic No. 21

Density and Specific Gravity

- **Density** is defined as mass per unit volume.

  That is, Density: $\rho = \dfrac{m}{V}$ (kg/m^3)
- The reciprocal of density is the **specific volume** v, which is defined as volume per unit mass.

  That is, $v = \dfrac{V}{m} = \dfrac{1}{\rho}$
- For a differential volume element of mass dm and volume dV, density can be expressed as $\rho = \dfrac{dm}{dV}$.
- The density of a substance, in general, depends on temperature and pressure.

- The density of most gases is proportional to pressure and inversely proportional to temperature.
- Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible.
- At 20°C, for example, the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm, a change of just 0.5 percent.
- The density of liquids and solids depends more strongly on temperature than it does on pressure.
- At 1 atm, for example, the density of water changes from 998 kg/m^3 at 20°C to 975 kg/m^3 at 75°C, a change of 2.3 percent, which can still be neglected in many engineering analyses.
- Sometimes the density of a substance is given relative to the density of a well-known substance.
- Then it is called **specific gravity**, or **relative density**, and is defined as the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C, for which ρ_{H_2O} = 1000 kg/m^3).

  That is, Specific gravity: $\mathrm{SG} = \dfrac{\rho}{\rho_{H_2O}}$

Topic No. 22

Vapor Pressure and Cavitation-1

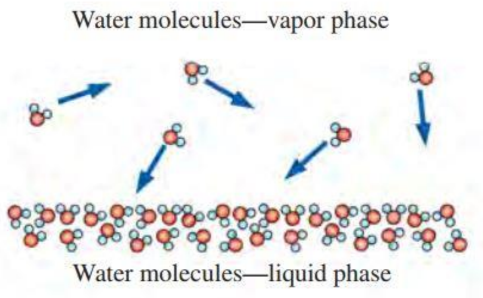
- It is well-established that temperature and pressure are dependent properties for pure substances during phase-change processes, and there is one-to-one correspondence between temperature and pressure.
- At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat}. Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure** P_{sat}.
- At an absolute pressure of 1 standard atmosphere (1 atm or 101.325 kPa), for example, the saturation temperature of water is 100°C.
- Conversely, at a temperature of 100°C, the saturation pressure of water is 1 atm.
- The **vapor pressure** P_v of a pure substance is defined as the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature.
- P_v is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{sat}$).

- We must be careful not to confuse vapor pressure with partial pressure. **Partial pressure** is defined as the pressure of a gas or vapor in a mixture with other gases.

Topic No. 23

Vapor Pressure and Cavitation-2

- For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor.
- The partial pressure of water vapor constitutes a small fraction (usually under 3 percent) of the atmospheric pressure since air is mostly nitrogen and oxygen.
- The partial pressure of a vapor must be less than or equal to the vapor pressure if there is no liquid present.
- However, when both vapor and liquid are present and the system is in phase equilibrium, the partial pressure of the vapor must equal the vapor pressure, and the system is said to be saturated.
- The rate of evaporation from open water bodies such as lakes is controlled by the difference between the vapor pressure and the partial pressure.
- The vapor bubbles (called **cavitation bubbles** since they form "cavities" in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.
- This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation**, and it is an important consideration in the design of hydraulic turbines and pumps.
- Cavitation must be avoided (or at least minimized) in most flow systems since it reduces performance, generates annoying vibrations and noise, and causes damage to equipment.
- The pressure spikes resulting from the large number of bubbles collapsing near a solid surface over a long period of time may cause erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery.
- The presence of cavitation in a flow system can be sensed by its characteristic tumbling sound.



- The vapor pressure (saturation pressure) of a pure substance (e.g., water) is the pressure exerted by its vapor molecules when the system is in phase equilibrium with its liquid molecules at a given temperature.

- Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 hours. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.

Topic No. 24

Viscosity-1

- When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion.
- To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force.
- The magnitude of the force needed to move the table depends on the friction coefficient between the table legs and the floor. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other.
- We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil.
- It appears that there is a property that represents the internal resistance of a fluid to motion or the “fluidity,” and that property is the **viscosity**.
- The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity.

- To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance ℓ.

Topic No. 25

Viscosity-2

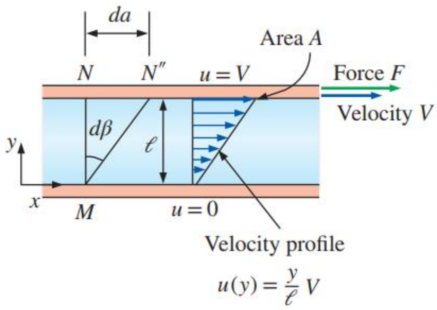
- Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant speed V.
- The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same speed, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A}$$

  where A is the contact area between the plate and the fluid. Note that the fluid layer deforms continuously under the influence of shear stress.
- The fluid in contact with the lower plate assumes the velocity of that plate, which is zero (because of the no-slip condition).
- In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and V, and thus the velocity profile and the velocity gradient are

$$u(y) = \frac{y}{\ell}V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$

  where y is the vertical distance from the lower plate.



Topic No. 26

Viscosity-3

- During a differential time interval dt, the sides of fluid particles along a vertical line MN rotate through a differential angle dβ while the upper plate moves a differential distance da = V dt.

- The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt$$

- Rearranging, the rate of deformation under the influence of shear stress τ becomes

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

- Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient du/dy.
- Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress τ ,

$$\tau \propto \frac{d\beta}{dt} \qquad \text{or} \qquad \tau \propto \frac{du}{dy}$$

- Fluids for which the rate of deformation is linearly proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687.
- Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.
- Blood and liquid plastics are examples of non-Newtonian fluids.
- In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

Shear stress: $$\tau = \mu \frac{du}{dy} \qquad (\text{N/m}^2)$$

- Where the constant of proportionality μ is called the **coefficient of viscosity** or the **dynamic** (or **absolute**) **viscosity** of the fluid, whose unit is kg/m·s, or equivalently, N·s/m^2 (or Pa·s where Pa is the pressure unit Pascal).
- A common viscosity unit is **poise**, which is equivalent to 0.1 Pa·s (or centipoise, which is one-hundredth of a poise). The viscosity of water at 20°C is 1.002 centipoise, and thus the unit centipoise serves as a useful reference.
- Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid,
  but deform continuously when the shear stress exceeds the yield stress and behave as a fluid.
- Such materials are referred to as Bingham plastics after Eugene C. Bingham (1878–1945), who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century.

Topic No. 27

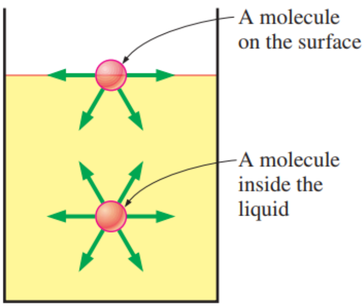
Viscosity-4

- In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently.
- For convenience, this ratio is given the name **kinematic viscosity** v and is expressed as v = μ / ρ.
- Two common units of kinematic viscosity are m^2/s and **stoke** (1 stoke = 1 cm^2/s = 0.0001 m^2/s).
- In general, the viscosity of a fluid depends on both temperature and pressure, although the dependence on pressure is rather weak.
- For liquids, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures.
- For gases, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure.
- The viscosity of a fluid is a measure of its resistance to the rate of deformation.
- Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.
- Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature.
- The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature.
- This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly.
- As a result, the energized liquid molecules can move more freely.
- In a gas, on the other hand, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities.
- This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow.
- Liquids, in general, are much more viscous than gases.

Topic No. 28

Surface Tension and Capillary Effect-1

- It is often observed that a drop of blood forms a hump on a horizontal glass; a drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface; water droplets from rain or dew hang from branches or leaves of trees; a liquid fuel injected into an engine forms a mist of spherical droplets; water dripping from a leaky faucet falls as nearly spherical droplets; a soap bubble released into the air forms a nearly spherical shape; and water beads up into small drops on flower petals.
- In these and other observances, liquid droplets behave like small balloons filled with the liquid, and the surface of the liquid acts like a stretched elastic membrane under tension.
- The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.
- The magnitude of this force per unit length is called **surface tension** or coefficient of surface tension σ_s and is usually expressed in the unit N/m (or lbf/ft in English units).
- This effect is also called surface energy (per unit area) and is expressed in the equivalent unit of $N \cdot m/m^2$ or J/m^2.
- In this case, σ_s represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.
- To visualize how surface tension arises, we present a microscopic view in Figure by considering two liquid molecules, one at the surface and one deep within the liquid body.

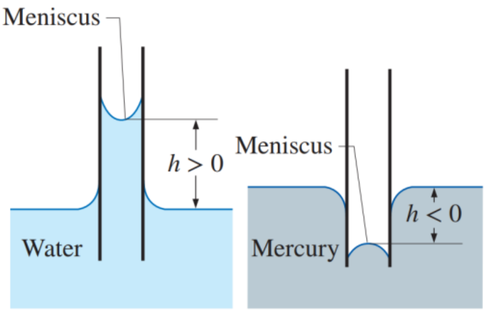


- The attractive forces applied on the interior molecule by the surrounding molecules balance each other because of symmetry.
- But the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small.
- Therefore, there is a net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid.

Topic No. 29

Surface Tension and Capillary Effect-2

- This force is balanced by the repulsive forces from the molecules below the surface that are trying to be compressed.
- The result is that the liquid minimizes its surface area.
- This is the reason for the tendency of liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.
- You also may have observed, with amusement, that some insects can land on water or even walk on water and that small steel needles can float on water.
- These phenomena are made possible by surface tension which balances the weights of these objects.
- The capillary effect is also partially responsible for the rise of water to the top of tall trees.
- The curved free surface of a liquid in a capillary tube is called the **meniscus**.
- This effect is usually expressed by saying that water wets the glass (by sticking to it) while mercury does not.
- The strength of the capillary effect is quantified by the **contact** (or wetting) **angle** ϕ, defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact.
- The surface tension force acts along this tangent line toward the solid surface.

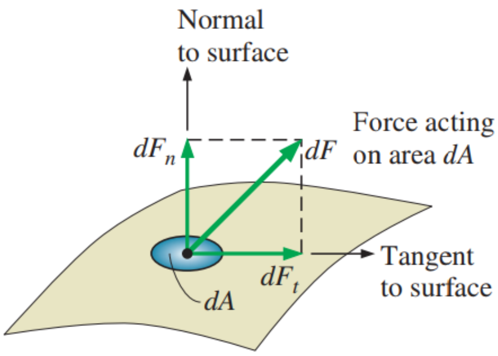


- A liquid is said to wet the surface when $\phi < 90°$ and not to wet the surface when $\phi > 90°$.
- In atmospheric air, the contact angle of water (and most other organic liquids) with glass is nearly zero, $\phi \approx 0°$.
- Therefore, the surface tension force acts upward on water in a glass tube along the circumference, tending to pull the water up.
- As a result, water rises in the tube until the weight of the liquid in the tube above the liquid level of the reservoir balances the surface tension force.
- The contact angle is 130° for mercury–glass and 26° for kerosene–glass in air.

Topic No. 30

Pressure and Stress

- **Pressure** is defined as a normal force exerted by a fluid per unit area.
- We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress.
- Since pressure is defined as force per unit area, it has units of newton per square meter (N/m^2), which engineers call a **Pascal** (Pa).
- That is, 1 Pa = 1 N/m^2
- The pressure unit Pascal is too small for most pressures encountered in practice. So units like kilo Pascal and mega Pascal are used appropriately.
- In the English system, the pressure unit is pound-force per square inch (lbf/in^2, or psi), and 1 atm = 14.696 psi.
- **Stress** is defined as force per unit area.
- Normal component: **normal stress.**
- In a fluid at rest, the normal stress is called **pressure**.
- Tangential component: **shear stress.**

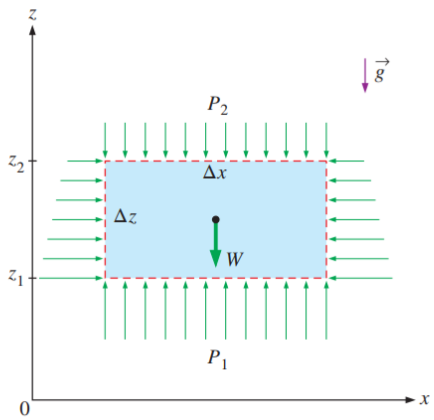


Topic No. 31

Variation of Pressure with Depth-1

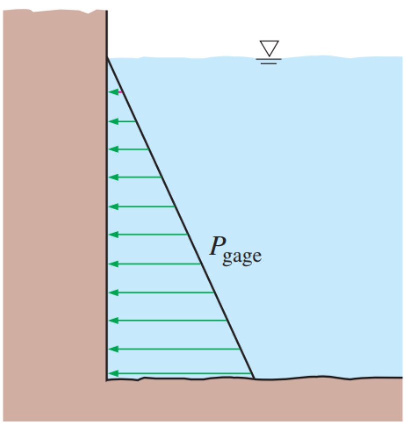
- Pressure in a fluid at rest does not change in the horizontal direction.
- This can be shown easily by considering a thin horizontal layer of fluid and doing a force balance in any horizontal direction.
- However, this is not the case in the vertical direction in a gravity field.
- Pressure in a fluid increases with depth because more fluid rests on deeper layers, and the effect of this “extra weight” on a deeper layer is balanced by an increase in pressure.

- To obtain a relation for the variation of pressure with depth, consider a rectangular fluid element of height Δz, length Δx, and unit depth (Δy = 1 into the page) in equilibrium.
- Assuming the density of the fluid ρ to be constant, a force balance in the vertical z-direction gives



Free-body diagram of a rectangular fluid element in equilibrium

$$\sum F_z = ma_z = 0$$
$$P_2\Delta x - P_1\Delta y - \rho g \Delta x \Delta z = 0$$
$$P = P_{atm} + \rho g h$$
$$P_{gage} = \rho g h$$



The pressure of a fluid at rest increases with depth (as a result of added weight).

- Thus, we conclude that the pressure difference between two points in a constant density fluid is proportional to the vertical distance Δz between the points and the density ρ of the fluid.

Topic No. 32

Variation of Pressure with Depth-2

- In other words, pressure in a static fluid increases linearly with depth.
- This is what a diver experiences when diving deeper in a lake.
- For a given fluid, the vertical distance Δz is sometimes used as a measure of pressure, and it is called the pressure head.
- We also know that for small to moderate distances, the variation of pressure with height is negligible for gases because of their low density.
- The pressure in a tank containing a gas, for example, can be considered to be uniform since the weight of the gas is too small to make a significant difference.
- Also, the pressure in a room filled with air can be approximated as a constant.

Topic No. 33

Variation of Pressure with Depth-3

- When the variation of density with elevation is known, the pressure difference between any two points 1 and 2 can be determined by integration to be

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g \; dz$$

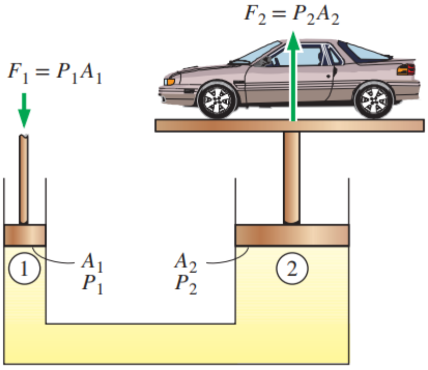
$$P_1 = P_2 \Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2}$$

Topic No. 34

Use of Pascal Law

- A consequence of the pressure in a fluid remaining constant in the horizontal direction is that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is called **Pascal's law**, after Blaise Pascal (1623–1662).
- Pascal also discovered that the force applied by a fluid is proportional to the surface area. He realized that two hydraulic cylinders of different areas could be connected, and the larger could be used to exert a proportionally greater force than that applied to the smaller.

- "Pascal's machine" has been the source of many inventions that are a part of our daily lives such as hydraulic brakes and lifts. This is what enables us to lift a car easily by one arm.



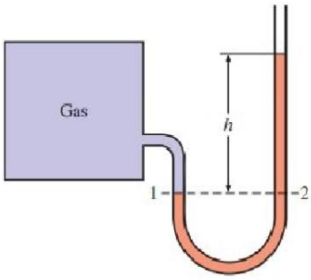
Topic No. 35

The Manometer

- We notice from equation:

$$\Delta P = P_2 - P_1 = -\rho g \Delta z = -\gamma_s \Delta z = -\int_1^2 \rho g \; dz$$

that an elevation change of -Δz in a fluid at rest corresponds to ΔP/ρ g, which suggests that a fluid column can be used to measure pressure differences. A device based on this principle is called a **manometer**, and it is commonly used to measure small and moderate pressure differences.



A simple U-tube manometer, with high pressure applied to the right side.

The basic manometer

- A manometer consists of a glass or plastic U-tube containing one or more fluids such as mercury, water, alcohol, or oil.
- To keep the size of the manometer to a manageable level, heavy fluids such as mercury are used if large pressure differences are anticipated.
- Consider the manometer shown in Figure that is used to measure the pressure in the tank. Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value. Furthermore, since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, $P_2 = P_1$.

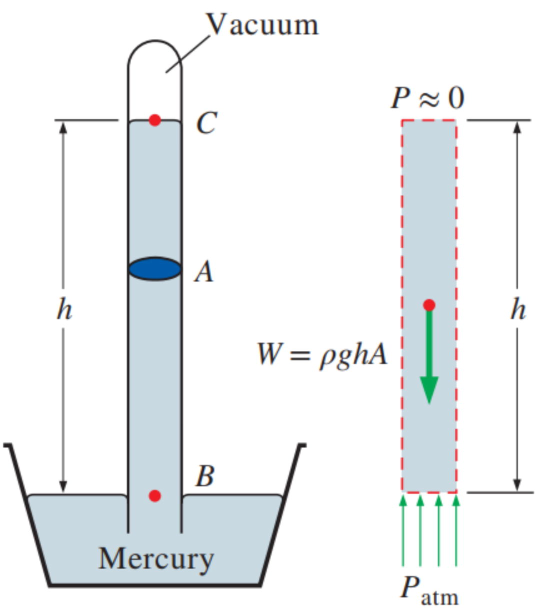
$$P_2 = P_{atm} + \rho gh$$

where ρ is the density of the manometer fluid in the tube.

Topic No. 36

The Barometer and Atmosphere Pressure

- Atmospheric pressure is measured by a device called a **barometer**; thus, the atmospheric pressure is often referred to as the barometric pressure.
- The pressure at point B is equal to the atmospheric pressure, and the pressure at point C can be taken to be zero since there is only mercury vapor above point C and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation.



- Writing a force balance in the vertical direction gives

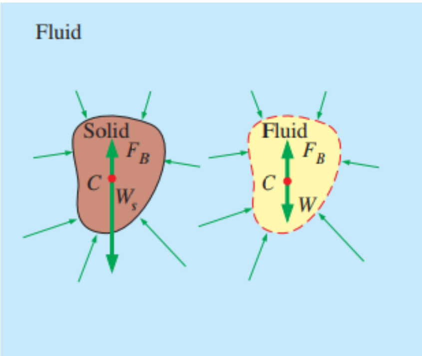
$$P_{atm} = \rho g h$$

where ρ is the density of mercury, g is the local gravitational acceleration, and h is the height of the mercury column above the free surface.
- Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer.
- A frequently used pressure unit is the standard atmosphere, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg} = 13,595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2).

Topic No. 37

Buoyancy and Stability

- It is a common experience that an object feels lighter and weighs less in a liquid than it does in air.
- This can be demonstrated easily by weighing a heavy object in water by a waterproof spring scale.
- Also, objects made of wood or other light materials float on water. These and other observations suggest that a fluid exerts an upward force on a body immersed in it.
- This force that tends to lift the body is called the **buoyant force** and is denoted by F_B.
- The buoyant force is caused by the increase of pressure in a fluid with depth.
- It may be noted that the buoyant force is independent of the distance of the body from the free surface.
- It is also independent of the density of the solid body.
- Consider an arbitrarily shaped solid body submerged in a fluid at rest and compare it to a body of fluid of the same shape indicated by dashed lines at the same distance from the free surface.



The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The buoyant force F_B acts upward through the centroid C of the displaced volume and is equal in magnitude to the weight W of the displaced fluid, but is opposite in direction. For a solid of uniform density, its weight W_s also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces. (Here $W_s > W$ and thus $W_s > F_B$; this solid body would sink.)

- The imaginary fluid body is in static equilibrium, and thus the net force and net moment acting on it are zero.
- Therefore, the upward buoyant force must be equal to the weight of the imaginary fluid body whose volume is equal to the volume of the solid body.
- Further, the weight and the buoyant force must have the same line of action to have a zero moment. This is known as **Archimedes' principle**, after the Greek mathematician Archimedes (287–212 BC).

Topic No. 38

Buoyancy and Stability

- The principle is expressed as "The buoyant force acting on a body of uniform density immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume."
- For floating bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body.

Stability of Immersed and Floating Bodies

- An important application of the buoyancy concept is the assessment of the stability of immersed and floating bodies with no external attachments.
- This topic is of great importance in the design of ships and submarines.

For floating bodies such as ships, stability is an important consideration for safety.

Topic No. 39

Fluid Kinematics: An Overview

- **Fluid Kinematics** deals with the motion of fluids without considering the forces and moments which create the motion.

➢ **Items discussed here:**

- Material derivative and its relationship to Lagrangian and Eulerian.
- Lagrangian and Eulerian descriptions of fluid flow.
- Flow visualization.
- Plotting flow data.
- Fundamental kinematics properties of fluid motion and deformation.
- Reynolds Transport Theorem

Topic No. 40

Acceleration Field

Consider a fluid particle and Newton's second law:

$$\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$$

The acceleration of the fluid particle is the time derivative of the particle's velocity:

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

However, particle velocity at a point is the same as the fluid velocity:

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{particle}, y_{particle}, z_{particle}, t)}{dt}$$

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{particle}}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y_{particle}}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z_{particle}}\frac{dz_{particle}}{dt}$$

$$\vec{a}_{particle}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Topic No. 41

Acceleration Field

In vector form:

$$\vec{a}_{particle}(x,y,z,t)=\frac{d\vec{V}}{dt}=\frac{\partial\vec{V}}{\partial t}+(\vec{V}.\vec{\nabla})\vec{V}$$

The Gradient or del operator:

$$\vec{\nabla}=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)=\vec{i}\,\frac{\partial}{\partial x}+\vec{j}\,\frac{\partial}{\partial y}+\vec{k}\,\frac{\partial}{\partial z}$$

And in Cartesian coordinates:

$$a_x=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$$

$$a_y=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}$$

$$a_z=\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}$$

- The first term in: $\vec{a}_{particle}(x,y,z,t)=\frac{d\vec{V}}{dt}=\frac{\partial\vec{V}}{\partial t}+(\vec{V}.\vec{\nabla})\vec{V}$

  $(\frac{\partial\vec{V}}{\partial t})$ is called the **local acceleration** and is nonzero only for unsteady flows.
- The second term ($(\vec{V}.\vec{\nabla})\vec{V}$) is called the **advective acceleration** and It accounts for the effect of the fluid particle moving (advecting or convecting) to a new location in the flow, where the velocity field is different.

Material Derivative

- The total derivative operator *d*/*dt* is called the **material derivative** and is often given the notation, *D*/*Dt*.

$$\frac{D}{Dt}=\frac{d}{dt}=\frac{\partial}{\partial t}+(\vec{V}.\vec{\nabla})$$

- Advective acceleration is nonlinear: source of many phenomenon and primary challenge in solving fluid flow problems.

- Provides "transformation" between Lagrangian and Eulerian Frames.
- Other names for the material derivative include **total**, **particle**, **Lagrangian**, **Eulerian**, and **substantial derivative**.

Topic No. 42

Flow Visualization

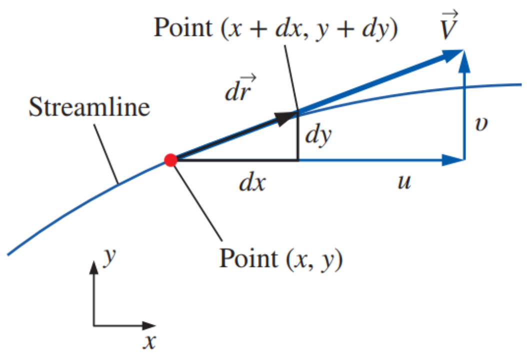
- Flow visualization is useful not only in physical experiments:
- Field features
- Important for both physical experiments and numerical [**computational fluid dynamics (CFD)**] solutions
- Numerous methods
- Streamlines and streamtubes
- Pathlines
- Streaklines
- Timelines
- Refractive techniques
- Surface flow techniques

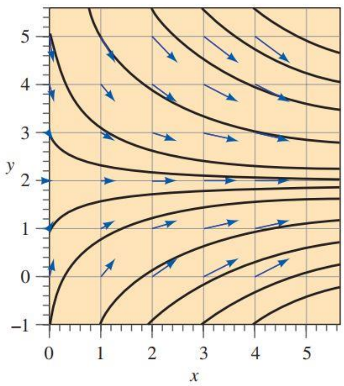
Topic No. 43

Streamlines

- A **streamline** is a curve that is everywhere tangent to the instantaneous local velocity vector.
- Consider an arc length $d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

  $d\vec{r}$ must be parallel to the local velocity vector $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$





- Geometric arguments results in the equation for a streamline as

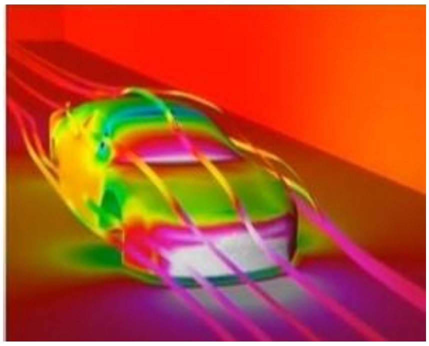
$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

- Streamline in the xy-plane

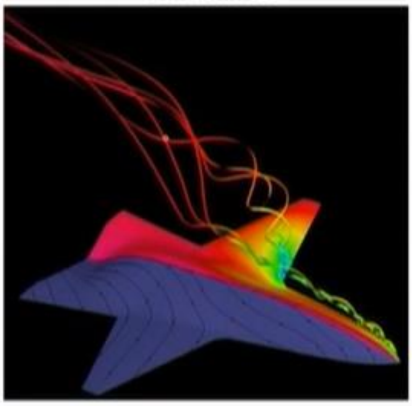
$$\left(\frac{dy}{dx}\right)_{along\ \ a\ streamline} = \frac{v}{u}$$

Streamlines: Examples

- NASCAR surface pressure contours and streamlines



- Airplane surface pressure contours, volume streamlines, and surface streamlines



Topic No. 44

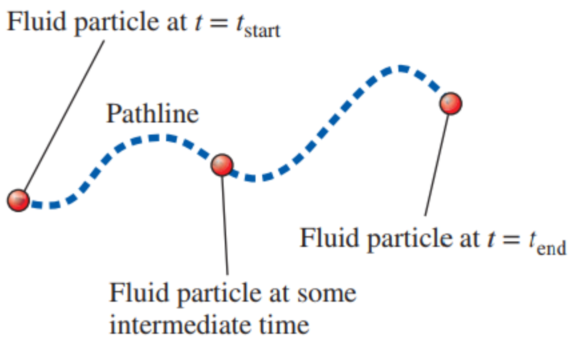
Pathlines

- A **pathline** is the actual path traveled by an individual fluid particle over some time period.
- Same as the fluid particles material position vector

$$\left(x_{particle}(t), y_{particle}(t), z_{particle}(t),\right)$$

- Particle location at time t:

$$x = x_{start} + \int_{t_{start}}^{1} V dt$$

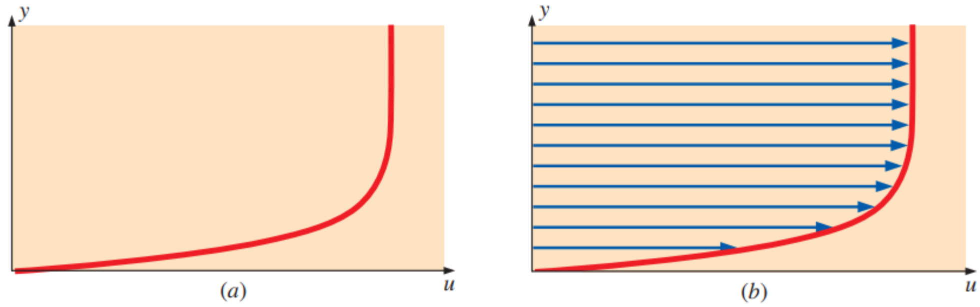


- **Particle image velocimetry (PIV)** is a modern experimental technique to measure velocity field over a plane in the flow field.

Topic No. 45

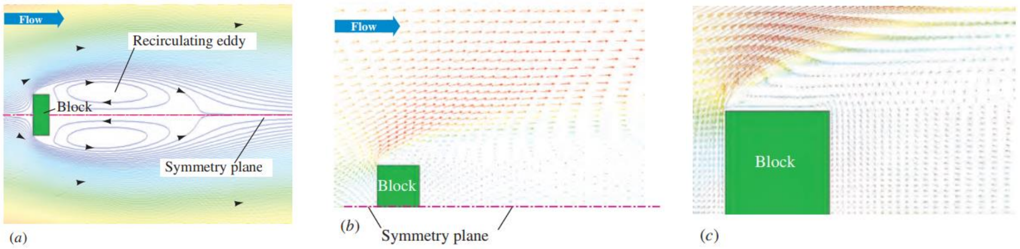
Plots of Data

- A **profile plot** indicates how the value of a scalar property varies along some desired direction in the flow field.



Standard profile plot and (*b*) Profile plot with arrows.

- A **vector plot** is an array of arrows indicating the magnitude and direction of a vector property at an instant in time.

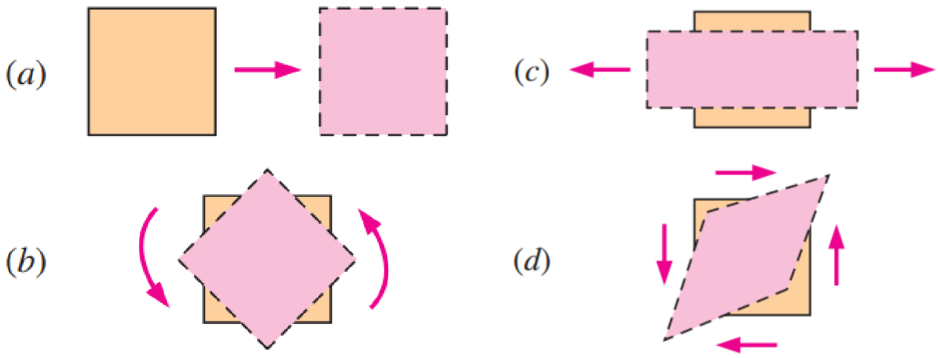


(*a*) streamlines, (*b*) velocity vector plot of the upper half of the flow, and (*c*) velocity vector plot, close up view revealing more details in the separated flow region.

Topic No. 46

Kinematics Description

- In fluid mechanics, an element may undergo four fundamental types of motion
  a) **Translation**
  b) **Rotation**
  c) **Linear strain** **(**sometimes called **extensional strain)**
  d) **Shear strain**.



- Because fluids are in constant motion, and deformation is best described in terms of rate.
- Velocity: rate of translation
- Angular velocity: rate of rotation
- Linear strain rate: rate of linear strain
- Shear strain rate: rate of shear strain

Topic No. 47

Rate of Translation and Rotation

- The **rate of translation vector** in Cartesian coordinates:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- **Rate of rotation (angular velocity)** at a point is defined as the average rotation rate of two initially perpendicular lines that intersect at that point.

- The **rate of rotation vector** is equal to the **angular velocity vector** and is expressed in Cartesian coordinates as

$$\vec{\omega} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

Linear Strain Rate

- **Linear strain rate** is defined as the rate of increase in length per unit length.

- **Linear strain rate** in Cartesian coordinates:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}, \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

- **Volumetric strain rate** in Cartesian coordinates:

$$\frac{1}{V}\frac{DV}{Dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

- The volumetric strain rate is zero in an incompressible flow.

Shear Strain Rate

- **Shear strain rate** at a point is defined as half of the rate of decrease of the angle between two initially perpendicular lines that intersect at the point.

- **Shear strain rate** can be expressed in Cartesian coordinates:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \ \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \ \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

- We can mathematically combine linear strain rate and shear strain rate into one symmetric second-order tensor called the **strain rate tensor.**

- **Strain rate tensor can** be expressed in Cartesian coordinates:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{pmatrix}$$

Topic No. 48

Shear Strain Rate

- Strain-rate tensor is important for numerous reasons. For example, to
- Develop relationships between fluid stress and strain rate.
- Extraction and flow visualization in CFD Simulations.

Vorticity and Rotationality

- The **vorticity vector** defined as the curl of the velocity vector $\vec{V}$.

$$\vec{\xi} = \vec{\nabla} \times \vec{V} = curl(\vec{V})$$

- The **rate of rotation vector** is equal to half of the vorticity vector.

$$\frac{1}{2}\vec{\nabla} \times \vec{V} = \frac{1}{2} curl(\vec{V}) = \frac{1}{2}\vec{\xi}$$

- **Vorticity** is equal to twice the angular velocity of a fluid particle.

- **Vorticity vector** in Cartesian coordinates:

$$\vec{\xi} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

- **Vorticity vector** in cylindrical coordinates:

$$\vec{\xi} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z$$

- In regions where $\vec{\xi} = 0$, the flow is called **irrotational**. Elsewhere the flow is called **rotational.**

Topic No. 49

Discussion of Shear Strain Rate

- Purpose of our discussion of fluid element kinematics:
- Better appreciation of the inherent complexity dynamics
- Mathematical sophistication required to fully describe fluid motion
- Understand the role of the material derivative in transforming between Lagrangian and Eulerian descriptions
- Distinguish between various types of flow visualization and methods of plotting the characteristics of a fluid flow
- Appreciate the many ways that fluids move and deform
- Distinguish between rotational and irrotational regions of flow based on the flow property vorticity
- Understand the usefulness of the Reynolds transport theorem.

Topic No. 050

Example of Steady Two-Dimensional Velocity Field

EXAMPLE 4–1 A Steady Two-Dimensional Velocity Field

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j} \qquad \textbf{(1)}$$

where the x- and y-coordinates are in meters and the magnitude of velocity is in m/s. A **stagnation point** is defined as *a point in the flow field where the velocity is zero.* (*a*) Determine if there are any stagnation points in this flow field and, if so, where? (*b*) Sketch velocity vectors at several locations in the domain between $x = -2$ m to 2 m and $y = 0$ m to 5 m; qualitatively describe the flow field.

SOLUTION For the given velocity field, the location(s) of stagnation point(s) are to be determined. Several velocity vectors are to be sketched and the velocity field is to be described.

Assumptions **1** The flow is steady and incompressible. **2** The flow is two-dimensional, implying no z-component of velocity and no variation of u or v with z.

Analysis (*a*) Since $\vec{V}$ is a vector, *all* its components must equal zero in order for $\vec{V}$ itself to be zero. Using Eq. 4–4 and setting Eq. 1 equal to zero,

Stagnation point:

$$u = 0.5 + 0.8x = 0 \quad \rightarrow \quad x = -0.625 \text{ m}$$
$$v = 1.5 - 0.8y = 0 \quad \rightarrow \quad y = 1.875 \text{ m}$$

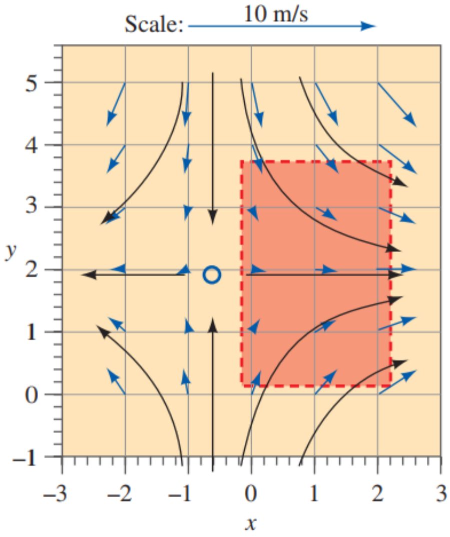
Yes. There is one stagnation point located at $\mathbf{x = -0.625}$ **m**, $\mathbf{y = 1.875}$ **m**.

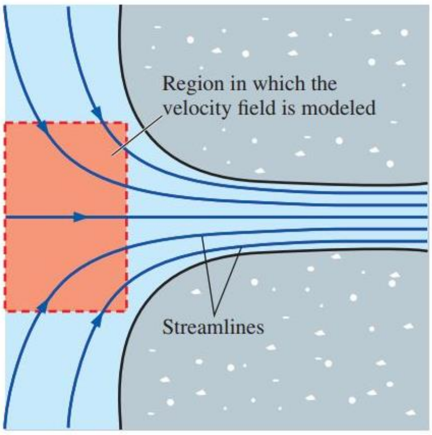
(*b*) The x- and y-components of velocity are calculated from Eq. 1 for several (x, y) locations in the specified range. For example, at the point ($x = 2$ m, $y = 3$ m), $u = 2.10$ m/s and $v = -0.900$ m/s. The magnitude of velocity (the *speed*) at that point is 2.28 m/s. At this and at an array of other locations, the velocity vector is constructed from its two components, the results of which are shown in Fig. 4–4. The flow can be described as stagnation point flow in which flow enters from the top and bottom and spreads out to the right and left about a horizontal line of symmetry at $y = 1.875$ m. The stagnation point of part (*a*) is indicated by the blue circle in Fig. 4–4.

Topic No. 51

A Steady Two-Dimensional Velocity Field

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$



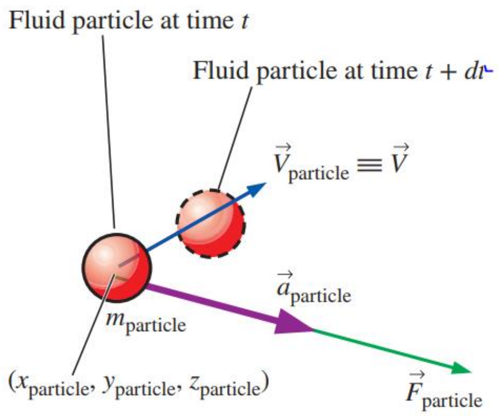


Velocity vectors (blue arrows) for the velocity field. The scale is shown by the top arrow, and the solid black curves represent the approximate shapes of some streamlines, based on the calculated velocity vectors. The stagnation point is indicated by the blue circle. The shaded region represents a portion of the flow field that can approximate flow into an inlet.

Topic No. 52

Acceleration Field

- The equations of motion for fluid flow (such as Newton's second law) are written for a fluid particle, which we also call a **material particle**.
- If we were to follow a particular fluid particle as it moves around in the flow, we would be employing the Lagrangian description, and the equations of motion would be directly applicable.
- For example, we would define the particle's location in space in terms of a **material position vector** ($x_{particle}(t)$, $y_{particle}(t)$, $z_{particle}(t)$).



Newton's second law: $\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$

Acceleration of a fluid particle: $\vec{a}_{particle} = \dfrac{d\vec{V}_{particle}}{dt}$

Velocity and Acceleration Field

$$\vec{V}_{particle}(t) = \vec{V}(x_{particle}(t),\ y_{particle}(t),\ z_{particle}(t),\ t)$$

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}}{dt} = \frac{d\vec{V}(x_{particle}, y_{particle}, z_{particle}, t)}{dt}$$

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{particle}}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y_{particle}}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z_{particle}}\frac{dz_{particle}}{dt}$$

$$\vec{a}_{particle}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

The Gradient or del operator:

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

Acceleration of a fluid particle expressed as a field variable:

$$\vec{a}_{particle}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}$$

The first term ($\frac{\partial \vec{V}}{\partial t}$) is called the **local acceleration** and is nonzero only for unsteady flows. The second term ($(\vec{V}.\vec{\nabla})\vec{V}$) is called the **advective acceleration** and is nonzero for steady and unsteady flows.

The components of the acceleration vector in Cartesian coordinates:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Local Acceleration Advective Acceleration

Topic No. 53

Examples of Acceleration Field

Flow of water through the nozzle of a garden hose illustrates that fluid particles may accelerate, even in a steady flow. In this example, the exit speed of the water is much higher than then water speed in the hose, implying that fluid particles have accelerated even though the flow is steady.

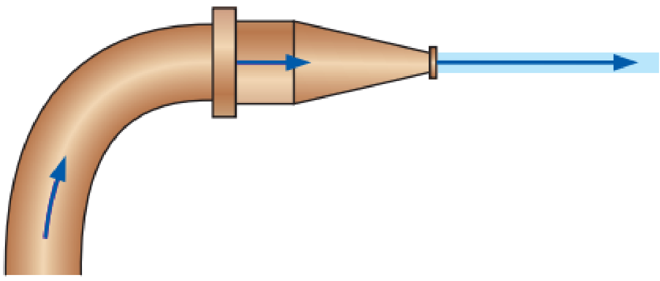


FIGURE 4–8

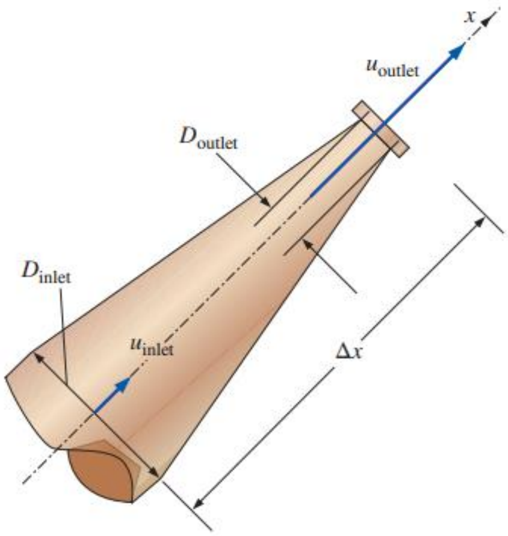


FIGURE 4–9
Flow of water through the nozzle of Example 4–2.

EXAMPLE 4–2 Acceleration of a Fluid Particle through a Nozzle

Nadeen is washing her car, using a nozzle similar to the one sketched in Fig. 4–8. The nozzle is 3.90 in (0.325 ft) long, with an inlet diameter of 0.420 in (0.0350 ft) and an outlet diameter of 0.182 in (see Fig. 4–9). The volume flow rate through the garden hose (and through the nozzle) is $\dot{V}$ = 0.841 gal/min (0.00187 ft^3/s), and the flow is steady. Estimate the magnitude of the acceleration of a fluid particle moving down the centerline of the nozzle.

SOLUTION The acceleration following a fluid particle down the center of a nozzle is to be estimated.

Assumptions **1** The flow is steady and incompressible. **2** The x-direction is taken along the centerline of the nozzle. **3** By symmetry, $v = w = 0$ along the centerline, but u increases through the nozzle.

Analysis The flow is steady, so you may be tempted to say that the acceleration is zero. However, even though the local acceleration $\partial\vec{V}/\partial t$ is identically zero for this steady flow field, the advective acceleration $(\vec{V}\cdot\vec{\nabla})\vec{V}$ is *not zero*. We first calculate the average x-component of velocity at the inlet and outlet of the nozzle by dividing volume flow rate by cross-sectional area:

Inlet speed:

$$u_{\text{inlet}} \cong \frac{\dot{V}}{A_{\text{inlet}}} = \frac{4\dot{V}}{\pi D_{\text{inlet}}^2} = \frac{4(0.00187\ \text{ft}^3/\text{s})}{\pi(0.0350\ \text{ft})^2} = 1.95\ \text{ft/s}$$

Similarly, the average outlet speed is $u_{\text{outlet}} = 10.4$ ft/s. We now calculate the acceleration in two ways, with equivalent results. First, a simple average value of acceleration in the x-direction is calculated based on the change in speed divided by an estimate of the **residence time** of a fluid particle in the nozzle, $\Delta t = \Delta x/u_{\text{avg}}$ (Fig. 4–10). By the fundamental definition of acceleration as the rate of change of velocity,

Method A: $$a_x \cong \frac{\Delta u}{\Delta t} = \frac{u_{\text{outlet}} - u_{\text{inlet}}}{\Delta x/u_{\text{avg}}} = \frac{u_{\text{outlet}} - u_{\text{inlet}}}{2\ \Delta x/(u_{\text{outlet}} + u_{\text{inlet}})} = \frac{u_{\text{outlet}}^2 - u_{\text{inlet}}^2}{2\ \Delta x}$$

The second method uses the equation for acceleration field components in Cartesian coordinates, Eq. 4–11,

Method B: $$a_x = \underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{Steady}}^{0} + u\frac{\partial u}{\partial x} + \underbrace{v\cancel{\frac{\partial u}{\partial y}}}_{v\,=\,0\text{ along centerline}}^{0} + \underbrace{w\cancel{\frac{\partial u}{\partial z}}}_{w\,=\,0\text{ along centerline}}^{0} \cong u_{\text{avg}}\frac{\Delta u}{\Delta x}$$

Here we see that only one advective term is nonzero. We approximate the average speed through the nozzle as the average of the inlet and outlet speeds, and we use a **first-order finite difference approximation** (Fig. 4–11) for the average value of derivative $\partial u/\partial x$ through the centerline of the nozzle:

$$a_x \cong \frac{u_{\text{outlet}} + u_{\text{inlet}}}{2}\,\frac{u_{\text{outlet}} - u_{\text{inlet}}}{\Delta x} = \frac{u_{\text{outlet}}^2 - u_{\text{inlet}}^2}{2\ \Delta x}$$

The result of method B is identical to that of method A. Substitution of the given values yields

Axial acceleration:

$$a_x \cong \frac{u_{\text{outlet}}^2 - u_{\text{inlet}}^2}{2\ \Delta x} = \frac{(10.4\ \text{ft/s})^2 - (1.95\ \text{ft/s})^2}{2(0.325\ \text{ft})} = \mathbf{160\ ft/s^2}$$

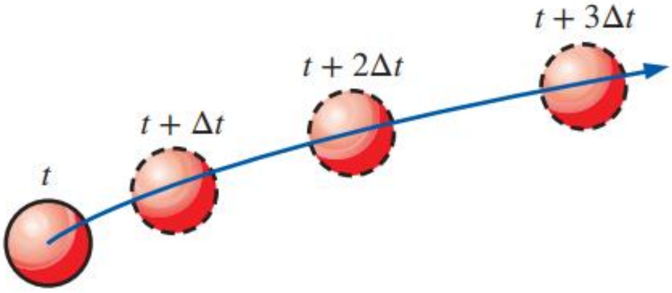
Discussion Fluid particles are accelerated through the nozzle at nearly five times the acceleration of gravity (almost five g's)! This simple example clearly illustrates that the acceleration of a fluid particle can be nonzero, even in steady flow. Note that the acceleration is actually a **point function**, whereas we have estimated a simple average acceleration through the entire nozzle.

Topic No. 54

Material Derivative

- The total derivative operator d/dt in following equation (1) is given a special name, the **material derivative**; it is assigned a special notation, D/Dt, in order to emphasize that it is formed by following a fluid particle as it moves through the flow field.
- Other names for the material derivative include **total**, **particle**, **Lagrangian**, **Eulerian**, and **substantial derivative**.

$$\vec{a}_{particle}(x,y,z,t)=\frac{d\vec{V}}{dt}=\frac{\partial\vec{V}}{\partial t}+(\vec{V}.\vec{\nabla})\vec{V} \qquad \text{......................(1)}$$



- The material derivative *D/Dt* is defined by following a fluid particle as it moves throughout the flow field. In this illustration, the fluid particle is accelerating to the right as it moves up and to the right.

Topic No. 55 & 56

Material Derivative Note from Example 1-1

$$\vec{V}=\ (u,v)=(0.5+0.8x)\vec{i}+(1.5-0.8y)\vec{j}$$

EXAMPLE 4–3 Material Acceleration of a Steady Velocity Field

Consider the steady, incompressible, two-dimensional velocity field of Example 4–1. (*a*) Calculate the material acceleration at the point (x = 2 m, y = 3 m). (*b*) Sketch the material acceleration vectors at the same array of x- and y-values as in Example 4–1.

SOLUTION For the given velocity field, the material acceleration vector is to be calculated at a particular point and plotted at an array of locations in the flow field.

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 0 + \overbrace{(0.5 + 0.8x)(0.8)} + \overbrace{(1.5 - 0.8y)(0)} + 0 = (0.4 + 0.64x)\ \text{m/s}^2$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 0 + \overbrace{(0.5 + 0.8x)(0)} + \overbrace{(1.5 - 0.8y)(-0.8)} + 0 = (-1.2 + 0.64y)\ \text{m/s}^2$$

At the point ($x = 2$ m, $y = 3$ m), $\boldsymbol{a_x = 1.68}$ **m/s²** and $\boldsymbol{a_y = 0.720}$ **m/s²**.

(*b*) The equations in part (*a*) are applied to an array of x- and y-values in the flow domain within the given limits, and the acceleration vectors are plotted in Fig. 4–14.

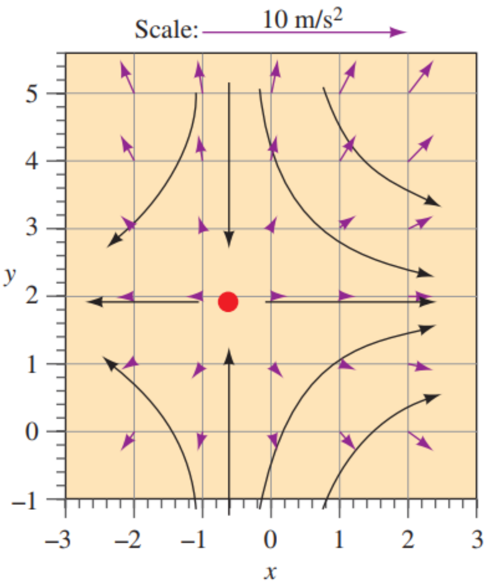


FIGURE 4–14

Topic No. 57

Velocity and Rotationability -1

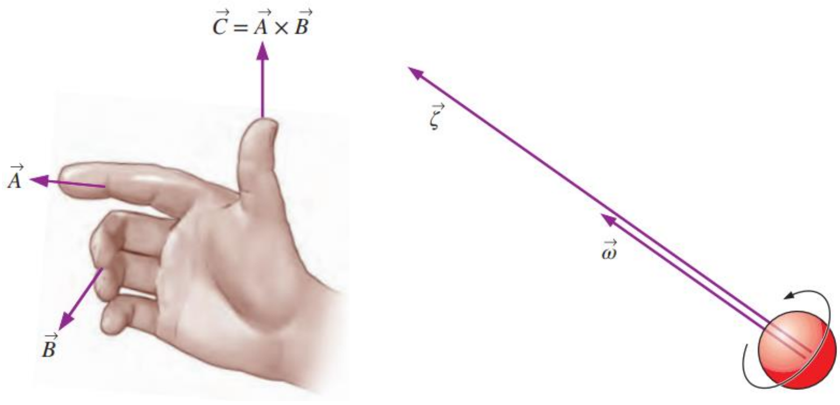
Another kinematic property of great importance to the analysis of fluid flows is the **vorticity vector**, defined mathematically as the curl of the velocity vector.

$$\vec{\xi} = \vec{\nabla} \times \vec{V} = curl(\vec{V})$$

The **rate of rotation vector** is equal to half of the vorticity vector.

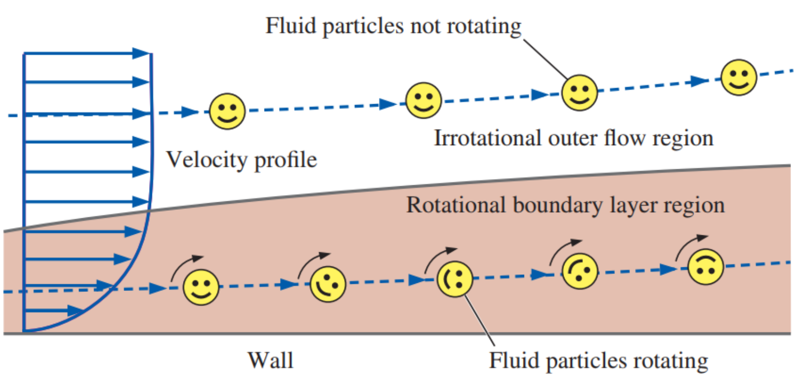
$$\frac{1}{2}\vec{\nabla} \times \vec{V} = \frac{1}{2}curl(\vec{V}) = \frac{1}{2}\vec{\xi}$$

Vorticity is equal to twice the angular velocity of a fluid particle.



The direction of a vector cross product is determined by the right-hand rule.

- If the vorticity at a point in a flow field is nonzero, the fluid particle that happens to occupy that point in space is rotating; the flow in that region is called **rotational**. ($\xi \neq 0$; **rotational**)
- Likewise, if the vorticity in a region of the flow is zero (or negligibly small), fluid particles there are not rotating; the flow in that region is called **irrotational**. ($\xi = 0$; **irrotational**)
- Physically, fluid particles in a rotational region of flow rotate end over end as they move along in the flow.
- The difference between rotational and irrotational flow: fluid elements in a rotational region of the flow rotate, but those in an irrotational region of the flow do not.



Topic No. 58

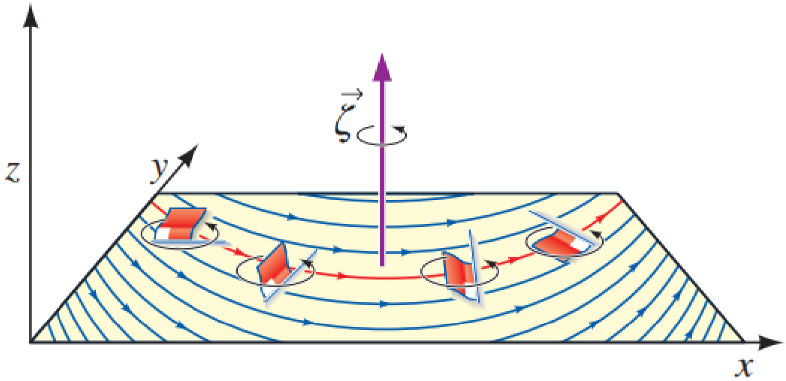
Velocity and Rotationability -2

- **Vorticity vector** in Cartesian coordinates:

$$\vec{\xi} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

- **Two-dimensional flow** in Cartesian coordinates:

$$\vec{\xi} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$



For two-dimensional flow in the *xy*-plane, the vorticity vector always points in the *z*- or -*z*-direction. In this illustration, the flag-shaped fluid particle rotates in the counterclockwise direction as it moves in the *xy*-plane; its vorticity points in the positive *z*-direction as shown.

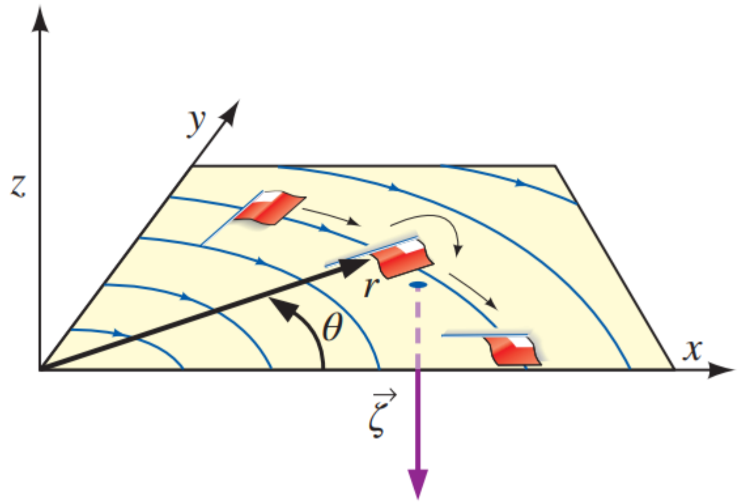
- **Vorticity vector** in cylindrical coordinates:

$$\vec{\xi} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{e}_z$$

- **Two-dimensional flow** in cylindrical coordinates:

$$\vec{\xi} = \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)\vec{k}$$

For a two-dimensional flow in the *rθ*-plane, the vorticity vector always points in the *z* (or -*z*) direction. In this illustration, the flag-shaped fluid particle rotates in the clockwise direction as it moves in the *rθ*-plane; its vorticity points in the -*z*-direction as shown.



Note: rotate clockwise = vorticity points in –z direction.
counter clockwise = +z direction.

Topic No. 59

Comparison of Two Circular Flows

Not all flows with circular streamlines are rotational.

Flow A-solid-body rotation:

$$u_r = 0 \quad \text{and} \quad u_\theta = \omega r$$

Flow B-line vortex:

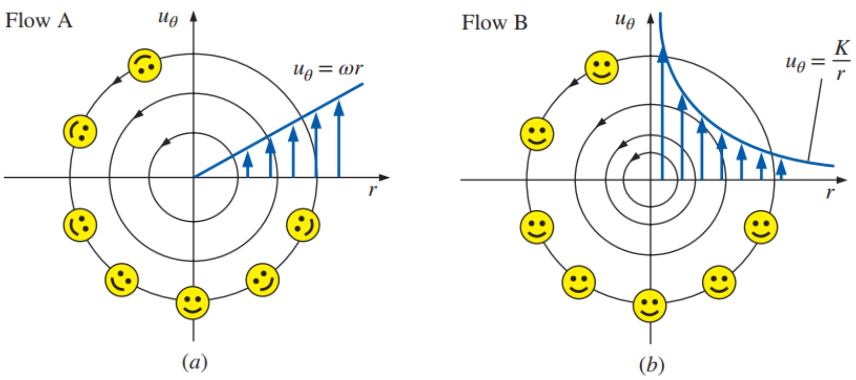
$$u_r = 0 \quad \text{and} \quad u_\theta = \frac{K}{r}$$

Flow A-solid-body rotation:

$$\vec{\xi} = \frac{1}{r}\left(\frac{\partial(\omega r^2)}{\partial r} - 0\right)\vec{k} = 2\omega\vec{k}$$

Flow B-line vortex:

$$\vec{\xi} = \frac{1}{r}\left(\frac{\partial(K)}{\partial r} - 0\right)\vec{k} = 0$$



Streamlines and velocity profiles for (a) flow A, solid-body rotation and (b) flow B, a line vortex. Flow A is rotational, but flow B is irrotational everywhere except at the origin. Note that the (oversized) fluid elements in flow B would also distort as they move, but in order to illustrate only particle rotation, such distortion is not shown here.

- A simple analogy can be made between flow A and a merry-go-round or roundabout, and flow B and a Ferris wheel.
- As children revolve around a roundabout, they also rotate at the same angular velocity as that of the ride itself.

(a)  (b) 

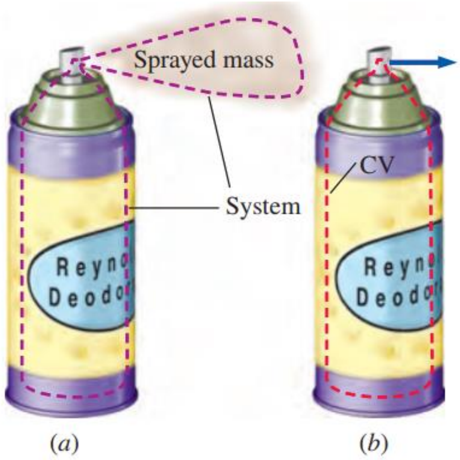
A simple analogy: (a) rotational circular flow is analogous to a roundabout, while (b) irrotational circular flow is analogous to a Ferris wheel.

Topic No. 60

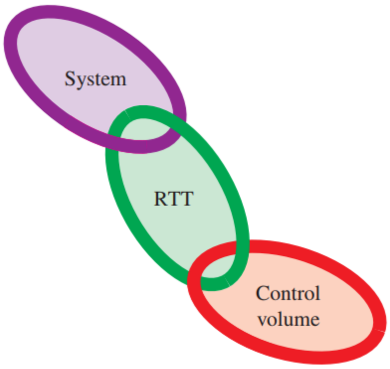
The Reynolds Transport Theorem

- Two methods of analyzing the spraying of deodorant from a spray can:
- (a) We follow the fluid as it moves and deforms. This is the system approach- no mass crosses the boundary, and the total mass of the system remains fixed.

- (b) We consider a fixed interior volume of the can. This is the control volume approach-mass crosses the boundary.



- The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the **Reynolds transport theorem (RTT)**.

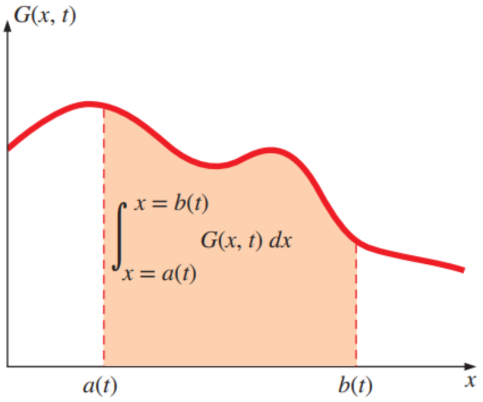


The Reynolds transport theorem (RTT) provides a link between the system approach and the control volume approach.

One-dimensional Leibniz theorem:

$$\frac{d}{dt}\int_{x=a(t)}^{x=b(t)} G(x,t)\ dx = \int_a^b \frac{\partial G}{\partial t}\ dx + \frac{db}{dt}G(b,t) - \frac{da}{dt}G(a,t)$$

- An elegant mathematical derivation of the Reynolds transport theorem is possible through use of the **Leibniz** (sometimes *Leibnitz*) **theorem.**
- The Leibniz theorem takes into account the change of limits *a*(*t*) and *b*(*t*) with respect to time, as well as the unsteady changes of integrand *G*(*x*, *t*) with time.



Topic No. 61

Application of Lebnitz Theorem

EXAMPLE 4–10 One-Dimensional Leibniz Integration

Reduce the following expression as far as possible:

$$F(t) = \frac{d}{dt}\int_{x=At}^{x=Bt} e^{-2x^2}\, dx$$

SOLUTION $F(t)$ is to be evaluated from the given expression.

Analysis The integral is

$$F(t) = \frac{d}{dt}\int_{x=At}^{x=Bt} e^{-2x^2}\, dx \tag{1}$$

We could try integrating first, and then differentiating, but we can instead use the 1-D Leibniz theorem. Here, $G(x, t) = e^{-2x^2}$ (G is not a function of time in this simple example). The limits of integration are $a(t) = At$ and $b(t) = Bt$. Thus,

$$F(t) = \int_a^b \frac{\partial G}{\partial t}\, dx + \frac{db}{dt} G(b, t) - \frac{da}{dt} G(a, t) \tag{2}$$

$$= \quad 0 \quad + Be^{-2b^2} \quad -Ae^{-2a^2}$$

or

$$F(t) = Be^{-2B^2t^2} - Ae^{-2A^2t^2} \tag{3}$$

Discussion You are welcome to try to obtain the same solution without using the Leibniz theorem.

In three dimensions, the Leibniz theorem for a *volume* integral is

Three-dimensional Leibniz theorem:

$$\frac{d}{dt}\int_{V(t)} G(x, y, z, t)\, dV = \int_{V(t)} \frac{\partial G}{\partial t}\, dV + \int_{A(t)} G\vec{V}_A \cdot \vec{n}\; dA \qquad \textbf{(4–50)}$$

where $V(t)$ is a moving and/or deforming volume (a function of time), $A(t)$ is its surface (boundary), and $\vec{V}_A$ is the absolute velocity of this (moving) surface (Fig. 4–63). Equation 4–50 is valid for *any* volume, moving and/or deforming arbitrarily in space and time. For consistency with the previous analyses, we set integrand G to ρb for application to fluid flow,

Three-dimensional Leibniz theorem applied to fluid flow:

$$\frac{d}{dt}\int_{V(t)} \rho b\, dV = \int_{V(t)} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{A(t)} \rho b\vec{V}_A \cdot \vec{n}\; dA \qquad \textbf{(4–51)}$$

If we apply the Leibniz theorem to the special case of a **material volume** (a system of fixed identity moving with the fluid flow), then $\vec{V}_A = \vec{V}$ everywhere on the material surface since it moves *with* the fluid. Here $\vec{V}$ is the local fluid velocity, and Eq. 4–51 becomes

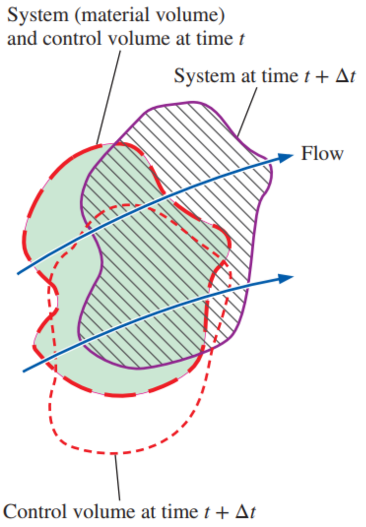
Leibniz theorem applied to a material volume:

$$\frac{d}{dt}\int_{V(t)} \rho b\, dV = \frac{dB_{sys}}{dt} = \int_{V(t)} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{A(t)} \rho b\vec{V} \cdot \vec{n}\; dA \qquad \textbf{(4–52)}$$

General RTT, non-fixed CV:

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\rho b) dV + \int_{CV} \rho b\vec{V}.\vec{n}\; dA$$

The equation above is valid for arbitrarily shaped, moving and/or deforming CV at time t. The material volume (system) and control volume occupy the same space at time t (the blue shaded area), but move and deform differently. At a later time they are not coincident.



EXAMPLE 4–11 Reynolds Transport Theorem in Terms of Relative Velocity

Beginning with the Leibniz theorem and the general Reynolds transport theorem for an arbitrarily moving and deforming control volume, Eq. 4–53, prove that Eq. 4–44 is valid.

SOLUTION Equation 4–44 is to be proven.

Analysis The general three-dimensional version of the Leibniz theorem, Eq. 4–50, applies to *any* volume. We choose to apply it to the control volume of interest, which can be moving and/or deforming differently than the material volume (Fig. 4–64). Setting G to ρb, Eq. 4–50 becomes

$$\frac{d}{dt}\int_{CV} \rho b \, dV = \int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV + \int_{CS} \rho b \vec{V}_{CS} \cdot \vec{n}\, dA \qquad (1)$$

We solve Eq. 4–53 for the control volume integral,

$$\int_{CV} \frac{\partial}{\partial t}(\rho b)\, dV = \frac{dB_{sys}}{dt} - \int_{CS} \rho b \vec{V} \cdot \vec{n}\, dA \qquad (2)$$

Substituting Eq. 2 into Eq. 1, we get

$$\frac{d}{dt}\int_{CV} \rho b \, dV = \frac{dB_{sys}}{dt} - \int_{CS} \rho b \vec{V} \cdot \vec{n}\, dA + \int_{CS} \rho b \vec{V}_{CS} \cdot \vec{n}\, dA \qquad (3)$$

Combining the last two terms and rearranging,

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b (\vec{V} - \vec{V}_{CS}) \cdot \vec{n}\, dA \qquad (4)$$

But recall that the relative velocity is defined by Eq. 4–43. Thus,

RTT in terms of relative velocity:
$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \int_{CS} \rho b \vec{V}_r \cdot \vec{n}\, dA \qquad (5)$$

Discussion Equation 5 is indeed identical to Eq. 4–44, and the power and elegance of the Leibniz theorem are demonstrated.

Topic No. 62

Exercise/Examples-1

Given that velocity $\vec{V} = (1+t)x\vec{i} + (2+t)y\vec{j}$

Find the equation of the

(a) Streamline

(b) Pathline and

(c) Streakline.

Given that the common point all three $x = 1,\ y = 2,\ z = 0$ at $t = 0$

Solution:

We observe

$$\vec{V} = (1+t)x\vec{i} + (2+t)y\vec{j}$$
$$\Rightarrow \begin{cases} u = (1+t)x \\ v = (2+t)y \end{cases}$$

(a) Streamline:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = ds$$

$$\begin{cases} \dfrac{dx}{u} = ds \\ \dfrac{dy}{v} = ds \end{cases} \Rightarrow \begin{cases} \dfrac{dx}{(1+t)x} = ds \\ \dfrac{dy}{(2+t)y} = ds \end{cases} \Rightarrow \begin{cases} \dfrac{dx}{x} = (1+t)ds \\ \dfrac{dy}{y} = (2+t)ds \end{cases}$$

$$\Rightarrow \begin{cases} \int \dfrac{dx}{x} = \int (1+t)ds \\ \int \dfrac{dy}{y} = \int (2+t)ds \end{cases} \Rightarrow \begin{cases} \ln x = (1+t)s + c_1(t) \\ \ln y = (2+t)s + c_2(t) \end{cases}$$

$$\Rightarrow \begin{cases} s = \dfrac{\ln x + c_1(t)}{(1+t)} \\ s = \dfrac{\ln y + c_2(t)}{(2+t)} \end{cases} \Rightarrow \frac{\ln x + c_1(t)}{(1+t)} = \frac{\ln y + c_2(t)}{(2+t)}$$

$$\Rightarrow (2+t)\ln x + (2+t)c_1(t) = (1+t)\ln y + (1+t)c_2(t)$$

$$\Rightarrow (2+t)\ln x = (1+t)\ln y + \left[(1+t)c_2(t) - (2+t)c_1(t)\right]$$

$$\Rightarrow \ln x^{(2+t)} = \ln y^{(1+t)} + c_3(t)$$

$$\Rightarrow \ln \frac{x^{(2+t)}}{y^{(1+t)}} = \ln c_4(t)$$

$$\Rightarrow \frac{x^{(2+t)}}{y^{(1+t)}} = c_4(t)$$

Initial Condition $x=1,\ y=2,\ z=0$ at $t=0$

$$\frac{1}{2} = c_4(t)$$

$$\Rightarrow \frac{x^{(2+t)}}{y^{(1+t)}} = \frac{1}{2}$$

$$\Rightarrow y^{(1+t)} = 2x^{(2+t)}$$

$$\Rightarrow y = \left[2x^{(2+t)}\right]^{\frac{1}{(1+t)}}$$

Topic No. 63

Exercise/Examples-2

(b) Pathline:

$$\begin{cases} u = \dfrac{dx}{dt} \\ v = \dfrac{dy}{dt} \end{cases} \Rightarrow \begin{cases} (1+t)x = \dfrac{dx}{dt} \\ (2+t)y = \dfrac{dy}{dt} \end{cases} \Rightarrow \begin{cases} (1+t)dt = \dfrac{dx}{x} \\ (2+t)dt = \dfrac{dy}{y} \end{cases}$$

$$\Rightarrow \begin{cases} \int (1+t)dt = \int \dfrac{dx}{x} \\ \int (2+t)dt = \int \dfrac{dy}{y} \end{cases} \Rightarrow \begin{cases} t + \dfrac{1}{2}t^2 = \ln x + c_1 \ldots\ldots\ldots\ldots\ldots(1) \\ 2t + \dfrac{1}{2}t^2 = \ln y + c_2 \ldots\ldots\ldots\ldots(2) \end{cases}$$

Initial Condition $x=1,\ y=2,\ z=0$ at $t=0$

$$c_1 = 0$$
$$c_2 = -\ln 2$$
$$\Rightarrow \begin{cases} t + \frac{1}{2}t^2 = \ln x + 0 \\ 2t + \frac{1}{2}t^2 = \ln y - \ln 2 \end{cases} \Rightarrow \begin{cases} t + \frac{1}{2}t^2 = \ln x \ldots\ldots\ldots\ldots\ldots(3) \\ 2t + \frac{1}{2}t^2 = \ln y/2 \ldots\ldots\ldots\ldots(4) \end{cases}$$

Subtracting equation (3) from equation (4), we have

$$t = \ln y/2 - \ln x \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

By equation (3) and equation (5)

$$\ln y/2 - \ln x + \frac{1}{2}\left(\ln y/2 - \ln x\right)^2 = \ln x$$
$$\ln y/2 - 2\ln x + \frac{1}{2}\left(\ln y/2 - \ln x\right)^2 = 0$$

Topic No. 64

Exercise/Examples-3

(C) Streakline:

$$\begin{cases} u = \frac{dx}{dt} \\ v = \frac{dy}{dt} \end{cases} \Rightarrow \begin{cases} (1+t)x = \frac{dx}{dt} \\ (2+t)y = \frac{dy}{dt} \end{cases} \Rightarrow \begin{cases} (1+t)dt = \frac{dx}{x} \\ (2+t)dt = \frac{dy}{y} \end{cases}$$

$$\Rightarrow \begin{cases} \int_{\xi}^{t}(1+t)dt = \int_{1}^{x}\frac{dx}{x} \\ \int_{\xi}^{t}(2+t)dt = \int_{2}^{y}\frac{dy}{y} \end{cases} \Rightarrow \begin{cases} \left[t + \frac{1}{2}t^2\right]_{\xi}^{t} = [\ln x]_1^x \\ \left[2t + \frac{1}{2}t^2\right]_{\xi}^{t} = [\ln y]_2^y \end{cases}$$

$$\Rightarrow \begin{cases} t + \frac{1}{2}t^2 - \xi - \frac{1}{2}\xi^2 = \ln x - \ln 1 \\ 2t + \frac{1}{2}t^2 - \xi - \frac{1}{2}\xi^2 = \ln y - \ln 2 \end{cases}$$

From equation (1) and equation (2)

$$\begin{cases} t+\frac{1}{2}t^2 = \ln x + c_1 \\ 2t+\frac{1}{2}t^2 = \ln y + c_2 \end{cases}$$

$x=1,\ y=2,\ z=0 \qquad \xi < t$

$$\Rightarrow \begin{cases} \xi+\frac{1}{2}\xi^2 = \ln 1 + c_1 \\ 2\xi+\frac{1}{2}\xi^2 = \ln 2 + c_2 \end{cases}$$

$$\Rightarrow \begin{cases} c_1 = \xi+\frac{1}{2}\xi^2 \\ c_2 = 2\xi+\frac{1}{2}\xi^2 - \ln 2 \end{cases}$$

Subsituting c_1 and c_2 into equation (1) and equation (2)

$$\begin{cases} t+\frac{1}{2}t^2 = \ln x + \xi + \frac{1}{2}\xi^2 \\ 2t+\frac{1}{2}t^2 = \ln y + 2\xi + \frac{1}{2}\xi^2 - \ln 2 \end{cases}$$

$$\Rightarrow \begin{cases} t+\frac{1}{2}t^2 = \ln x + \xi + \frac{1}{2}\xi^2 \text{.................}(6) \\ 2t+\frac{1}{2}t^2 = \ln \frac{y}{2} + 2\xi + \frac{1}{2}\xi^2 \text{................}(7) \end{cases}$$

Subtracting equation (6) from (7)

$$t = \ln \frac{y}{2} - \ln x + \xi$$

$$\Rightarrow \xi = t - \ln \frac{y}{2} + \ln x$$

$$\Rightarrow \xi = t + \ln \frac{2x}{y} \text{...................}(8)$$

From equation (6) and (8), we have

$$t+\frac{1}{2}t^2 = \ln x + t + \ln\frac{2x}{y} + \frac{1}{2}\left(t+\ln\frac{2x}{y}\right)^2$$

$$\Rightarrow \frac{1}{2}t^2 = \ln x + \ln\frac{2x}{y} + \frac{1}{2}\left(t^2 + 2t\ln\frac{2x}{y} + \left(\ln\frac{2x}{y}\right)^2\right)$$

$$\Rightarrow \frac{1}{2}t^2 = \ln x + \ln\frac{2x}{y} + \frac{1}{2}t^2 + t\ln\frac{2x}{y} + \frac{1}{2}\left(\ln\frac{2x}{y}\right)^2$$

$$\Rightarrow \ln x + \ln\frac{2x}{y} + t\ln\frac{2x}{y} + \frac{1}{2}\left(\ln\frac{2x}{y}\right)^2 = 0$$

For $t = 0$

$$\Rightarrow \ln x + \ln\frac{2x}{y} + \frac{1}{2}\left(\ln\frac{2x}{y}\right)^2 = 0$$

Topic No. 65

Conservation of Mass

- The conservation of mass principle is one of the most fundamental principles in nature.
- We are all familiar with this principle, and it is not difficult to understand.
- If we mix 1 g of oil with 2 g of vinegar, then we know how much dressing of oil and vinegar we are going to obtain.
- Similarly 1 kg of oxygen reacts with 2 kg of hydrogen, 1 kg water is formed.
- All this contributes to understand the idea of mass conservation.
- In an electrolysis process, the water separates back to 2 kg of hydrogen and 1 kg of oxygen.
- Likewise energy, is a conserved property, and it cannot be created or destroyed during a process.
- However, mass m and energy E can be converted to each other according to the well-known formula proposed by Albert Einstein (1879–1955), $E = mc^2$ where c is the speed of light in a vacuum.
- This equation suggests that the mass of a system changes when its energy changes.
- However, for all energy interactions encountered in practice, with the exceptions of nuclear reactions, the change in mass is extremely small and cannot be detected by even the most sensitive device.
- For example, when 1 kg of water is formed from oxygen and hydrogen, the amount of energy released is 15,879 kJ which corresponds to a mass of 1.76×10^{-10} kg.

- This mass is so small that it can be ignored without any loss of accuracy. Thus we have,
- Laws of conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

- Similarly laws for conservation of Momentum say that momentum is preserved and
- Laws of conservation of energy:

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{CV}}{dt}$$

Topic No. 66

Mass, Bernoulli and Energy Equations

- Winds turbine “farms” are being constructed all over the world to extract kinetic energy from the wind and convert it to electrical energy.
- The mass, energy, momentum, and angular momentum balances are utilized in the design of a wind turbine. The Bernoulli equation is also useful in the preliminary design stage.

- Already familiar with **conservation laws** such as the laws of
- Conservation of mass,
- Conservation of energy, and
- Conservation of momentum.
- Traditionally, the conservation laws are first applied to a fixed quantity of matter called a closed system or just a system, and then extended to regions in space called control volumes.

Many fluid flow devices such as this Pelton wheel hydraulic turbine are analyzed by applying the conservation of mass and energy principles, along with the linear momentum equation.

Conservation of Mass

- The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys} = constant or $dm_{sys}/dt = 0$, which is the statement that the mass of the system remains constant during a process.
- Mass balance for a control volume (CV) in rate form :

- **Conservation of mass:**

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

- $\dot{m}_{in}$ and $\dot{m}_{out}$ is the total rates of mass flow into and out of the control volume.
- $\frac{dm_{CV}}{dt}$ is the rates of change of mass within the control volume boundries.
- In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the continuity equation.

Topic No. 67

The Linear Momentum Equation

- **Linear momentum:** The product of the mass and the velocity of a body is called the linear momentum or just the momentum of the body.
- The momentum of a rigid body of mass m moving with a velocity V is mV.

- Newton's second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.
- **Conservation of momentum principle:** The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such systems is conserved.
- **Linear momentum equation:** In fluid mechanics, Newton's second law is usually referred to as the linear momentum equation.
- **The conservation energy principle (**the energy balance): The net energy transfer to or from a system during a process be equal to the change in the energy content of the system.
- Energy can be transferred to or from a closed system by heat or work.
- Control volumes also involve energy transfer via mass flow.

- **Conservation of energy:**

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{CV}}{dt}$$

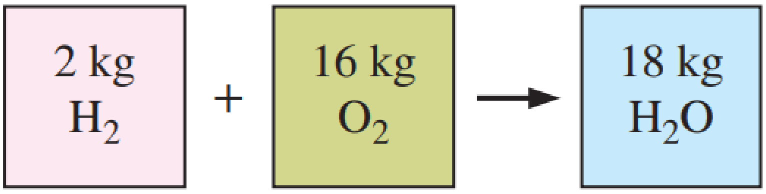
- $\dot{E}_{in}$ and $\dot{E}_{out}$ is the total rates of energy transfer into and out of the control volume.
- $\frac{dm_{CV}}{dt}$ is the rate of change of energy within the control volume boundaries.

Note: In fluid mechanics, we usually limit our consideration to mechanical forms of energy only.

Topic No. 68

Conservation of Mass

- **Conservation of mas:** Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.
- **Closed systems:** The mass of the system remain constant during a process.
- **Control volumes:** Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



Mass is conserved even during chemical reactions.

Mass and Volume Flow Rates

- **Mass flow rate:** The amount of mass flowing through a cross section per unit time.

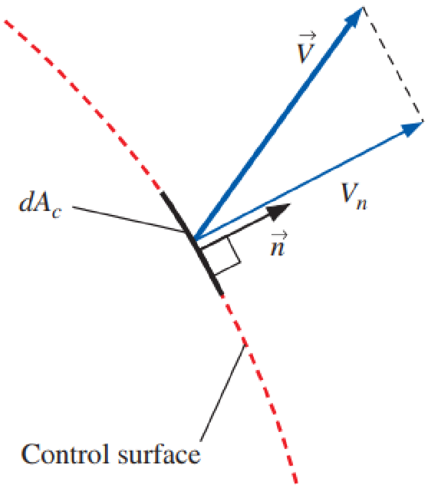
  The differential **mass flow rate:**

$$\delta \dot{m} = \rho V_n dA_c$$

Point functions have exact differentials.

$$\int_1^2 dA_c = A_{c2} - A_{c1} = \pi(r_2^{\ 2} - r_1^{\ 2}) \text{ but } \int_1^2 \delta \dot{m} = \dot{m}_{total} \text{ not } \dot{m}_2 - \dot{m}_1$$

Path functions have inexact differentials.



The normal velocity V_n for a surface is the component of velocity perpendicular to the surface.

Average velocity:

$$V_{avg} = \frac{1}{A_c}\int_{A_c} V_n d\dot{A}_c$$

Mass flow rate:

$$\delta \dot{m} = \rho V_n dA_c \qquad (kg/s)$$

Volume flow rate:

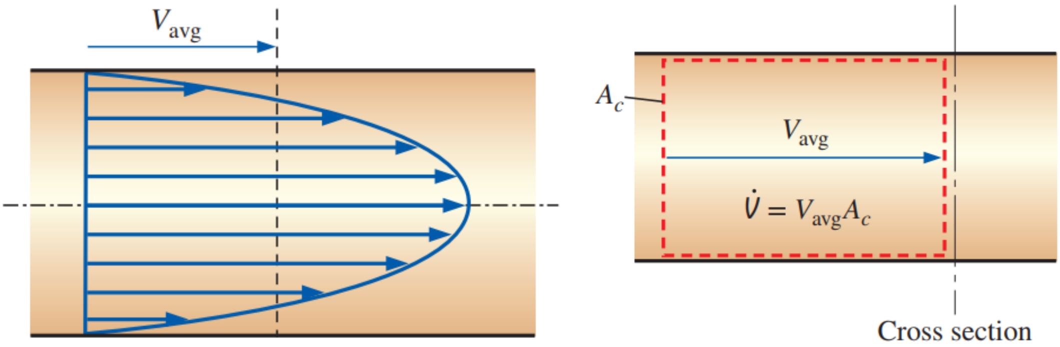
$$\dot{V} = \int_{A_c} V_n dA_c = V_{avg} A_c = VA_c \qquad (m^3/s)$$

The mass and volume flow rates are related by

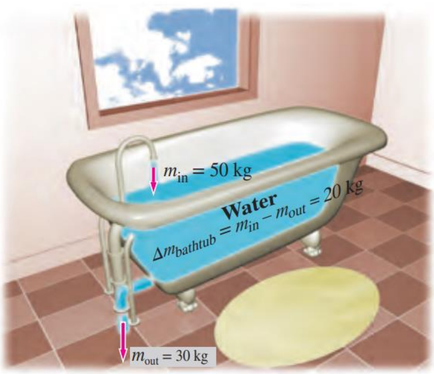
$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$

Topic No. 69

Mass and Volume Flow Rates

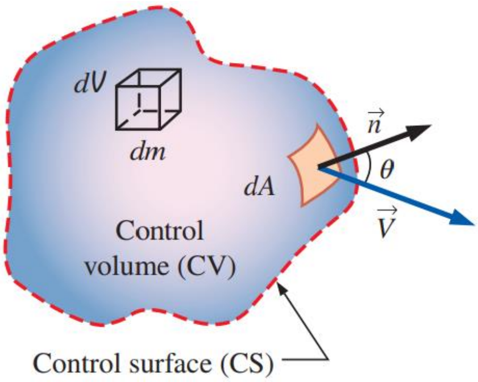


- Average velocity V_{avg} is defined as the average speed through a cross section.
- The volume flow rate is the volume of fluid flowing through a cross section per unit time.



Conservation of mass principle for an ordinary bathtub.

Mass balance and are applicable to any control volume undergoing any kind of process.



The differential control volume *dV* and the differential control surface *dA* used in the derivation of the conservation of mass relation.

- **General conservation of mass:**

$$\frac{d}{dt}\int_{CV}\rho dV+\int_{CV}\rho(\vec{V}.\vec{n})dA=0$$

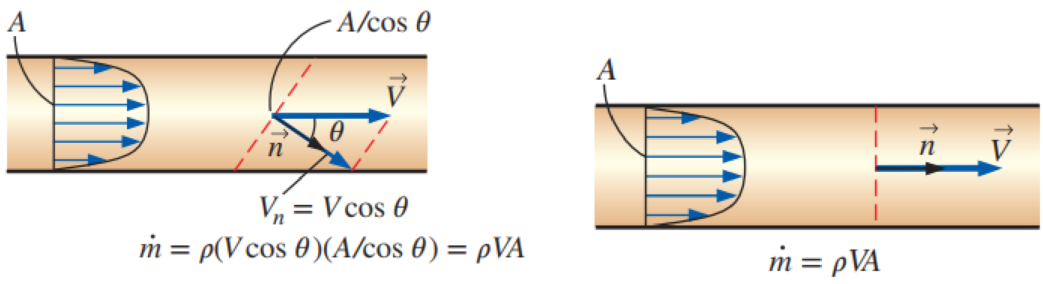
- The time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

The general conservation of mass relation can also be expressed as

$$\frac{d}{dt}\int_{CV}\rho dV+\sum_{out}\rho|V_n|A-\sum_{in}\rho|V_n|A=0$$

Using the definition of mass flow rate can also be expressed as

$$\frac{d}{dt}\int_{CV}\rho dV=\sum_{in}\dot{m}-\sum_{out}\dot{m}\quad\text{or}\quad\frac{dm_{CV}}{dt}=\sum_{in}\dot{m}-\sum_{out}\dot{m}$$



(*a*) Control surface *at an angle* to the flow (*b*) Control surface *normal* to the flow

- A control surface should always be selected normal to the flow at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b\, dV + \int_{CS} \rho b(\vec{V}\cdot\vec{n})\, dA$$

$$B = m \qquad b = 1 \qquad b = 1$$

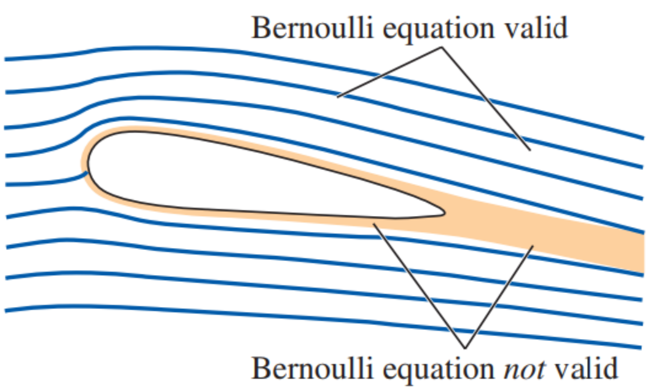
$$\frac{dm_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

The conservation of mass equation is obtained by replacing B in the Reynolds transport theorem by mass m, and b by 1 (m per unit mass = 1).

Topic No. 70

Bernoulli Equation

- **Bernoulli equation:** An approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.
- Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.
- The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The Bernoulli equation is an approximate equation that is valid only in inviscid regions of flow where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of boundary layers and wakes.

Acceleration of a Fluid Particle

- In two-dimensional flow, the acceleration can be decomposed into two components: streamwise acceleration a_s along the streamline and normal acceleration a_n in the direction normal to the streamline, which is given as $a_n = V^2 / R$
- Streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction.
- For particles that move along a straight path, $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

- Mathematically, we can be expressed as follows: We take the velocity *V* of a fluid particle to be a function of *s* and *t*. Taking the total differential of *V*(*s, t*)

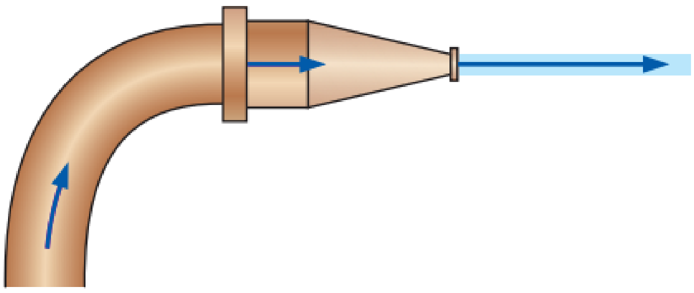
$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \qquad \text{and} \qquad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

In steady flow $\partial V / \partial t = 0$ and thus $V = V(s)$, and the acceleration in the *s*-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V\frac{dV}{ds}$$

Where $V = ds / dt$

- Acceleration in steady flow is due to the change of velocity with position.

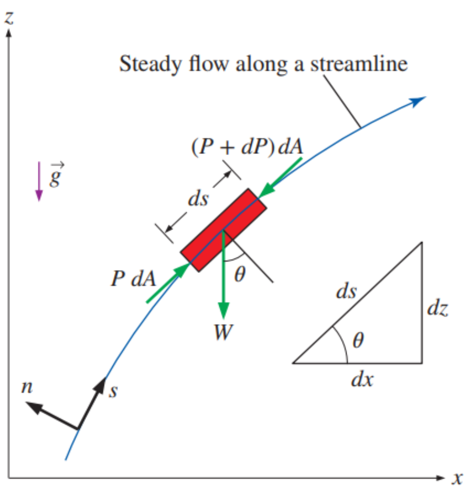


During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

Topic No. 71

Derivation of Bernoulli Equation-1

- The forces acting on a fluid particle along a streamline.
- The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.



$$P\,dA-(P+dP)dA-W\sin\theta=mV\frac{dV}{ds}\text{.............(1)}$$

$$m=\rho V=\rho\,dA\,ds$$

$$W=mg=\rho g\,dA\,ds$$

$$\sin\theta={dz}/{ds}$$

Subsituting equation (1)

$$-dP\,dA-\rho g\,dA\,ds\frac{dz}{ds}=\rho\,dA\,ds\,V\frac{dV}{ds}$$

$$\Rightarrow -dP-\rho g\,dz=\rho V\,dV$$

Put $V\,dV=\frac{1}{2}d(V^2)$

$$\Rightarrow \frac{dP}{\rho}+\frac{1}{2}d(V^2)+g\,dz=0$$

Integrating, we have

$$\int\left(\frac{dP}{\rho}+\frac{1}{2}d(V^2)+g\,dz\right)=\int 0$$

Steady flow:

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a stream line)}$$

Steady, incompressible flow:

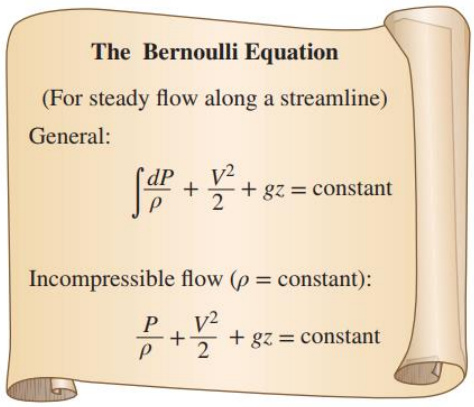
$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a stream line)}$$

- The Bernoulli equation can also be written between any two points on the same streamline as

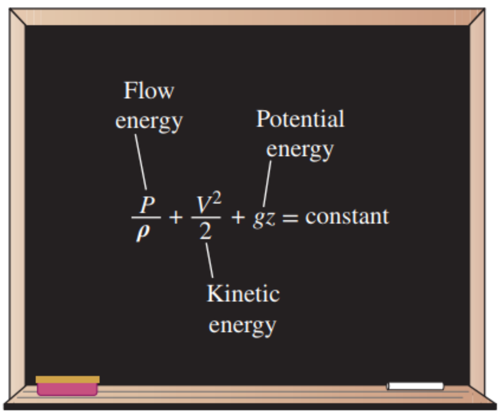
Steady, incompressible flow: $$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Topic No. 72

Derivation of Bernoulli Equation-2



- The incompressible Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.



The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

- The Bernoulli equation can be viewed as the "conservation of mechanical energy principle."
- This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately.
- The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.
- There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.

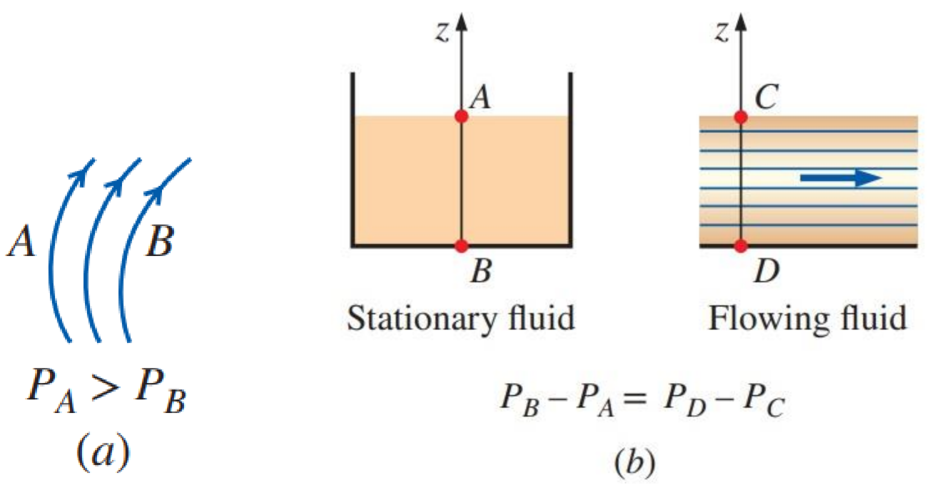
Topic No. 73

Force Balance across Streamlines

- Force balance in the direction *n* normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

$$\frac{P}{\rho}+\int\frac{V^2}{R}dn+gz=\text{constant} \quad (\text{across streamlines})$$

- For flow along a straight line, $R\to\infty$ and this equation reduces to $P/\rho+gz=\text{constant}$ or $P=-\rho gz+\text{constant}$, which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body.



- Pressure decreases towards the center of curvature when streamlines are curved (*a*), but the variation of pressure with elevation in steady, incompressible flow along a straight line (*b*) is the same as that in stationary fluid.

Unsteady, Compressible Flow

- The Bernoulli equation for unsteady, compressible flow:

Unsteady compressible flow: $$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = \text{ constant}$$

Topic No. 74

Static, Dynamic, and Stagnation Pressure

- The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density ρ.

$$\rho\left(\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a stream line)}\right)$$

$$\Rightarrow P + \rho\frac{V^2}{2} + \rho gz = \text{ constant (along a streamline)}$$

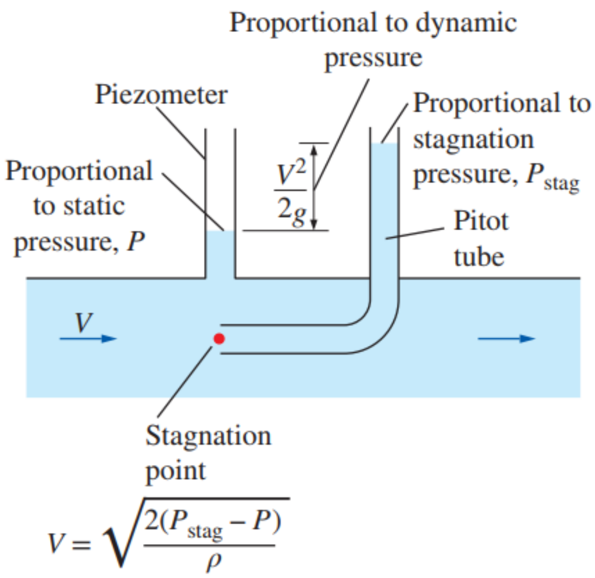
- P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.
- $\rho V^2/2$ is the **dynamic pressure**; it represents the pressure rise when the fluid in motion is brought to a stop isentropically.
- ρgz is the **hydrostatic pressure** term, which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., fluid weight on pressure. (Be careful of the sign—unlike hydrostatic pressure ρgh which increases with fluid depth h, the hydrostatic pressure term ρgz decreases with fluid depth.)
- Note that the sum of the static, dynamic, and hydrostatic pressures is called the **total pressure**.
- Therefore, the Bernoulli equation states that the total pressure along a streamline is constant.

- **Stagnation pressure:** The sum of the static and dynamic pressures is called the **stagnation pressure**. and it is expressed as

$$P_{stag} = P + \rho \frac{V^2}{2} \qquad \text{(kPa)}$$

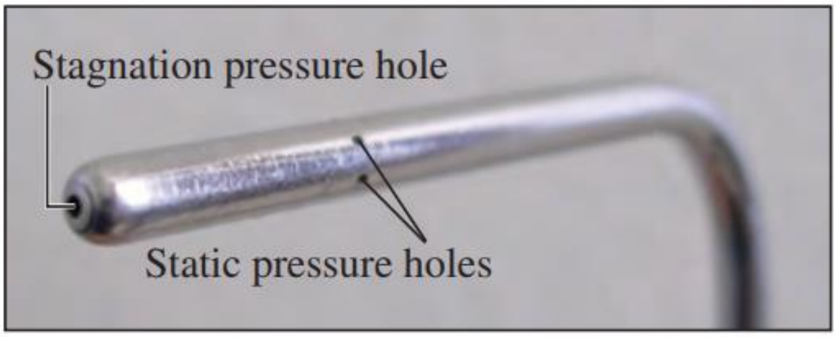
- The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically.
- The fluid velocity at that location is calculated from

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

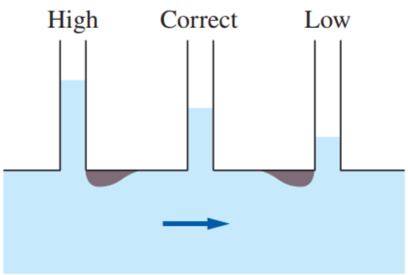


The static, dynamic, and stagnation pressure heads measured

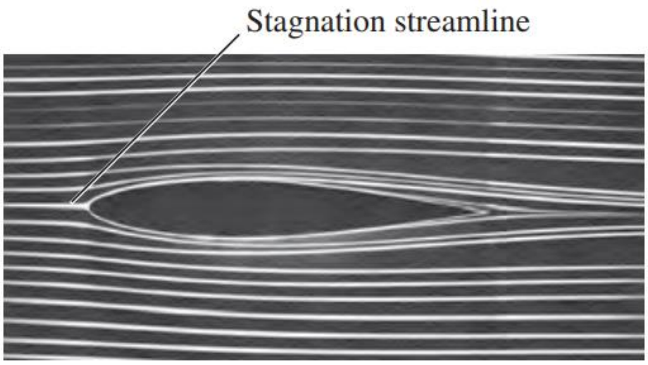
using piezometer tubes.



Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.



Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.



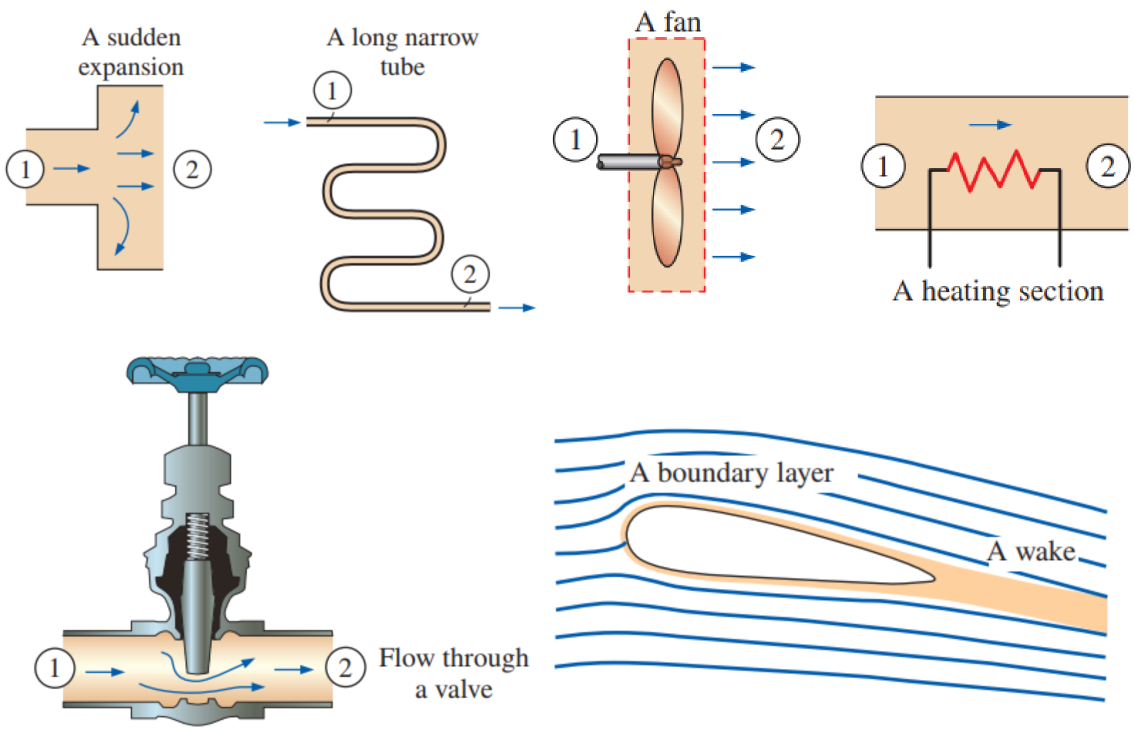
Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady; the streaklines are the same as streamlines and pathlines. The stagnation streamline is marked.

Topic No. 75

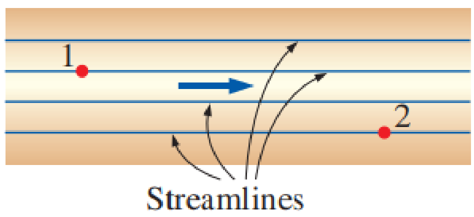
Limitations on the Use of the Bernoulli Equation

- **Steady flow:** The first limitation on the Bernoulli equation is that it is applicable to *steady flow*.
- **Frictionless flow/Negligible viscous effects:** Every flow involves some friction, no matter how small, and frictional effects may or may not be negligible.
- **No shaft work:** The Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles.
- When these devices, the energy equation should be used instead.
- **Incompressible flow:** Density is taken constant in the derivation of the Bernoulli equation. The flow is incompressible for liquids and also by gases at Mach numbers less than about 0.3.

- **No/Negligible heat transfer:** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.
- **Flow along a streamline:** Strictly speaking, the Bernoulli equation $P/\rho + V^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C is generally different for different streamlines.
- However, when a region of the flow is *irrotational* and there is no *vorticity* in the flow field, the value of the constant C remains the same for all streamlines, and the Bernoulli equation becomes applicable *across* streamlines as well.



Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.



$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).